On Capacity of the Many-to-One Multi-Channel Multi-Radio Wireless Mesh Networks

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Abstract

This paper focuses on the analysis of the per-node throughput capacity of a many-to-one ring-based Multi-Channel Multi-Radio interface (MCMR) Wireless Mesh Networks (WMN). We firstly introduce the ring-based network model and obtain the minimum number of mesh nodes which guarantees the network connectivity. Interference constraint and destination bottleneck constraint are shown as two key factors which affect the throughput capacity and we obtained the upper capacity bound of ring-based MCMR WMN by analyzing these two constraints. The result shows that the capacity not only relies on the ratio of channel number $c$ and radio interfaces number $m$, but also closely relates with guard parameter $δ$ and the number of rings $k$. In summary, in many-to-one ring-based MCMR WMN with $n$ mesh routers and each node is equipped with $c$ channels and $m$ radio interfaces, the per-node throughput capacity is shown to be $O(\min\{W, \frac{m}{\delta k}, \frac{Wm}{cn\delta^2}\})$ bits/sec.

Keywords: Many-to-One Networks; Wireless Mesh Networks; Throughput Capacity; Bottleneck Collision Area

1 Introduction

Wireless Mesh Network (WMN) gains so much popularity these days and is playing an important role in upcoming next generation Internet. Deriving from wireless ad hoc networks, WMN is originally designed to provide last mile connectivity to gateway nodes or access points. In [1], the authors gave a detailed comparison between WMN and wireless ad hoc networks. The differences are mainly focused on their mobility of nodes and overall network architecture. All nodes in wireless ad hoc networks are dynamic and equally communicate with each other. On the contrary, WMN often organized as a hierarchical architecture. For performing different functions, WMN has three kinds of nodes: gateway nodes, mesh routes and mesh clients, respectively. Gateway nodes are connected with the wired networks directly. Mesh routers are stationary and responsible for communications between mesh clients and gateway nodes. Mesh clients, however, are mobile and mainly communicate with mesh routers. Considering the differences above, the techniques used in wireless ad hoc networks can not be directly adopted in WMN.

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Both WMN and wireless ad hoc networks are worked in a multi-hop fashion. Packets may traverse multiple hops to reach their destinations. The nodes with single channel single interface possibly perform unsatisfactorily due to mutual interference and unfairness between the one-hop nodes and multi-hop nodes. In order to deal with these problems, some new techniques are developed, such as MCMR (Multi-Channel Multi-Radio), directional antenna, cognitive radios, cross-layer optimization, MIMO (Multiple-Input Multiple-Output) and so on.

With multi-channel multi-radio, some critical issues should be considered both in designing period and practical deployment stage, including network connectivity, minimal interference, improved aggregated network throughput, fairness and load balance. First of all, the network connectivity should be ensured. A node shares the same radio interface and channel with its neighbors in order to communicate with them. The network is likely to be partitioned by careless channel allocation scheme and this is devastating to both users and service providers. Second, the interference is tense especially when node density is high. Generally speaking, the mesh nodes are placed around the gateway nodes. The closer to gateway node, the more traffic load the node will take. There is a tradeoff between network connectivity and minimal interference. Thirdly, the aggregated network throughput should be improved compared to single channel single interface nodes. Last but not least, traffic load should be optimally routed along the shortest path while ensuring that one-hop nodes do not consume all available bandwidth.

Faced with the above problems, an analytical prediction on both network capacity and per-node capacity should be observed in advance, which provides useful guidance in practical deployment. It motivates us to explore the detailed relationship among the number of interfaces, the number of channels and the number of gateways. This paper mainly concentrates on throughput analysis in many-to-one WMN with each node having \( m \) radio interfaces and operating on \( c \) non-overlapping channels simultaneously. The analysis concludes that the per-node capacity in many-to-one wireless mesh networks with \( k \) rings is at most \( O(\min(\delta, \sqrt{W / \delta}), \sqrt{W / \delta}) \) bits/sec. This result provides guidance in designing process of wireless mesh networks.

2 Related Work

Gupta and Kumar gave an asymptotic analysis to capacity of wireless networks with \( n \) nodes and concluded that the network capacity is to be \( O(\log n) \) [2]. The contributions of this work provide useful reference to capacity analysis on WMN. Investigations show that, aggregated throughput capacity of the whole network is increased either by using multiple channels on a single radio interface, or by using multiple channels multiple radio interfaces on each node. In first case, however, having one interface and splitting a channel into several sub-channels for each mesh node does not help to improve the overall capacity [2].

When each node is equipped with multiple channels multiple radio interfaces, abbreviated as MCMR network, the situation would be more complicated. The number of channels and interfaces are critical parameters that would greatly impact on network capacity and throughput. In [3], the author showed that the network capacity is \( \Theta(\sqrt{1/(n \lg n)}) \), where \( n \) is the number of nodes, under the condition that the number of channels is \( O(\log n) \) when nodes are equipped with multi-channels and multi-interfaces [4] gave a detailed capacity analysis and experimental study with multi-interface and multi-channel, and proposed a multi-interface routing protocol (MIRP). The influence of multiple gateways was also investigated in this paper. Comparing to [3] and [4],
our work concentrates on the analytical throughput capacity analysis of many-to-one ring-based network.

In [4] and [5], the traffic of end-users received by mesh routers was divided into two categories: the one through gateway nodes and the other to end-users within the same mesh coverage area, which relate to infrastructure throughput and pure ad hoc throughput respectively. Therefore, the throughput capacity bound is derived by combining the capacity bound of these two different types of traffic. Pure ad hoc throughput has the same capacity bound as common wireless multi-channel networks. By practical observation, we know that infrastructure throughput is remarkably larger than pure ad hoc throughput, especially among backbone mesh routers. Mesh routers in many-to-one ring-based network model form the backbone of whole network. The many-to-one ring-based network model is highly characterized by its traffic pattern that the gateway node is the destination of many sources. For this consideration, we focus on the capacity analysis of infrastructure throughput in this paper. On the other hand, differing from [4], the network model in our work assumes that all gateway nodes do not produce communication traffic. But both mesh clients and mesh routers produce traffic, and mesh routers are also responsible for relaying the traffic which originates from mesh client nodes within their coverage area.

In [6], the author studied the capacity of multi-hop nodes by pre-allocating some link capacity to one-hop nodes and investigated the extent to which the capacity reserved for nodes that are two or more hops away can be used efficiently. The nodes that are directly connected with the gateway node are required to be fair between its own traffic and relaying one. The work provides us useful insight on considering the fairness between one-hop nodes and the other nodes in WMN.

The result of capacity analysis is affected by the specific definition of bottleneck collision area. In [5], the transmission area which is occupied by transmission to the receiving node in the network disk domain is minimized to one quarter of the area of disk of radius \( \frac{\delta}{2} \) times hop length. It lays the foundation to compute the bottleneck collision area. In [6], in order to prevent the hidden node problem, the collision area is required to be \((3 + \delta) d_{\max}\), where \( d_{\max} \) is the maximum link length and \( \delta \) is the guard zone parameter. In [7], the bottleneck collision area is defined as a circle centered at the gateway node with diameter \((q + 1)r\), where \( qr \) is the interference range. In [8], the bottleneck collision area is used and defined as the circle centered at the base station, and with radius \( d_{\text{reuse}}/2 \), where \( d_{\text{reuse}} \) is defined as \( 2p + f \), and \( p \) is the distance between transmission node and receiving node, \( f \) is the interference range. And the upper bound is shown to closely relate with the ratio between transmission range and network radius. In our analysis, the collision area is defined as \( \delta/2 \) times the length of the hop around each receiver.

3 Network Model and Definitions

3.1 Network Model

The network can be either flat or hierarchical and the latter is more commonly used. Two traditional wireless network models are tree-based model and ring-based model. The tree-based architecture is hard to implement due to the distributed characteristic of WMN. Therefore, the ring-based networks are fully studied. Our work focuses on many-to-one ring-based WMN. The whole network, as shown in Fig. 1, is organized as multiple circles centered at gateway node. Wireless mesh nodes are evenly distributed and mesh clients are mainly located at the outer
circle for they are incapable of relaying data packets. We assume that the node (except gateway) within \( i \)-th circle only can communicate with the nodes in adjacent rings. Every node has multiple channels and multiple radio interfaces.

![Ring-based network model](image)

Fig. 1: Ring-based network model. The transmission range is \( r \) and interference range \( \tilde{r} = (1 + \delta)r \). Here we assume \( \delta = 1 \)

The network model in this paper is denoted as \((n, m, c)\)-network, where \( n \) is the number of mesh routers, \( m \) is the number of radio interfaces in each router, and \( c \) is the number of available channels that each radio interface operates on. The transmission range \( r \) and interference range \( \tilde{r} \) of each node, as shown in Fig. 1, are assumed to be the same for all mesh routers. Since surplus interfaces will be a waste of resources due to mutual interference, we let \( m \leq c \). The channels used in this paper are assumed to be non-overlapping in nature and can be used simultaneously for transmission. Lots of factors lead to limited number of non-overlapping channels. IEEE 802.11a operates on 5 GHz band and has 13 non-overlapping channels, while IEEE 802.11 b/g works on 2.4 GHz band and has only 3 non-overlapping channels. Meanwhile, variable number of non-overlapping channels are designed in IEEE 8.2.11a standard for countries might use different frequency bands. Also, the number of channels is obviously constrained by radios deployed in practise.

Let the total capacity be \( W \) bits/sec. We assume that the rate of channel \( i \), \( 1 \leq i \leq c \), is assumed to be the same and is denoted as \( W_i \). Then we have

\[
W_i = W/c,
\]

and

\[
W = \sum_{i=1}^{c} W_i.
\]

The placement of nodes are not optimally distributed in practical applications. Thus, this paper firstly assumed that nodes are organized in a canonical topology, as shown in Fig. 2. The analysis result derived from canonical networks can be easily extended to general many-to-one ring-based networks. The nodes are located in rings centered at the gateway node. The width of each ring is the same as transmission range \( r \). Since the node in each ring can only communicate
with nodes which are located in adjacent rings, all nodes in each ring should cover the whole area of this ring to keep the network fully connected. Taken the outer most \( k \)-th ring for example, as shown in Fig. 3, the rings centered at the node \( u \) and the node \( v \) are joint in the \( k \)-th ring and the corresponding angle is \( \theta \). The number of nodes in the \( k \)-th ring, denoted as \( n_k \) \((k = 1, 2, \cdots)\), should meets the requirement that \( n_k \theta \geq 2\pi \). From Fig. 3, we have

\[
\cos\left(\frac{\theta}{2}\right) = \frac{(kr - \frac{1}{2}r)^2 + (kr)^2 - r^2}{2(kr - \frac{1}{2}r)(kr)} = 1 - \frac{3}{4k(2k - 1)}.
\]

Thus, \( \theta \) can be expressed as:

\[
\theta = 2 \arccos\left(1 - \frac{3}{4k(2k - 1)}\right).
\]
Combining Equation (1) and the requirement of network connectivity, we have:

\[ n_k \geq \frac{2\pi}{\theta} = \frac{\pi}{\arccos\left(1 - \frac{3}{4k(2k-1)}\right)}. \]  

(2)

Based on the assumption that the nodes are evenly distributed within the whole area, the number of nodes in the \( k \)-th ring is:

\[ n_k = \frac{2k-1}{k^2}n. \]  

(3)

From Equation (2) and (3), we can see that, in order to make sure the network is fully connected without being partitioned into subnetworks, following constraints should be meet when it comes to the total number of nodes \( n \):

\[ n \geq \frac{k^2\pi}{(2k-1)\arccos\left(1 - \frac{3}{4k(2k-1)}\right)}. \]  

(4)

In practise, the number of nodes are sufficiently large and satisfies the above requirement in Equation (4). In this paper, the above requirement on the total number of mesh nodes is assumed to be meet to maintain network connectivity.

We would like to mention that, even though the network model presented above assumes that there is only one gateway node, the result of capacity analysis in this paper can be extended to multiple gateway nodes networks with minor modification. At the same time, the result based on the canonical topology networks also applied to general wireless mesh networks.

### 3.2 Interference Model

According to [9], both protocol model and physical model can be used to capture the fundamentals for a successful transmission. In this paper, the protocol model is used and a transmission from node \( i \) to node \( j \) is successful if

\[ d(k, j) > (1 + \delta)d(i, j) \]

is true, where \( k \) is the node which simultaneously initiates a transmission using the same channel as \( i \) and \( j \). The parameter \( \delta \), called guard zone, is used to protect the node from interfering with each other. And \( d(x, y) \) is the distance between node \( x \) and node \( y \).

As we defined, the interference range, denoted as \( \tilde{r} \), is assumed to be the same for all mesh routers, and \( \tilde{r} = (1 + \delta)r \). Following most existing works, we let \( \delta = 1 \), thus, \( \tilde{r} = 2r \). Based on the interference model, the collision domain of a link \( l \), denoted as \( I(l) \), is defined as the link set which includes the \( i \)-th link itself and the links that should not be concurrently happen with the \( i \)-th link. The bottleneck collision area is defined as the geographical area of the network which is bounded by above the amount of data being transmitted in the network [10], which implies that transmission within the collision area can be concurrently done with non-overlapping channel or different radios.
4 Upper Bounds Capacity Analysis on Ring-Based MCMR WMNs

The throughput capacity analysis aims at finding out how capacity is changed with the increment of mesh routers and mesh client nodes. The analysis can be easily extended to multiple gateway nodes. In our many-to-one ring-based network topology, both mesh clients and mesh routers generate network traffic which is assumed to go through the gateway node. The link is supposed to be bi-directional between each node pair. The node, including mesh routers and mesh clients, are evenly distributed within an circular area with diameter of \(2kr\), where \(k\) is the number of rings \((0 \leq k)\) and \(r\) is the radius of the first ring. Initially, each node was equipped with omni-antenna for both receive and transmission. Taken the \(i\)-th ring for example, the number of nodes in \(i\)-th ring is defined as

\[
n_i = \frac{2i - 1}{k^2} n.
\]

Since the traffic goes up and down through gateway node, we logically organize mesh nodes as a tree. Each node within the \((i - 1)\)-th ring is the parent node of the \(i\)-th ring. Even though some node’s parent may fall on the same ring, the result is almost the same. Thus, the nodes in the \(i\)-th ring can be treated as \(i\) hops far from the gateway node. The sub-tree rooted at node \(v\) is denoted as \(T(v)\). The traffic load of link \(l(v)\), as shown in Fig. 4, is mainly determined by the number of node in \(T(v)\).

![Fig. 4: Subtree rooted at node \(v\)](image)

The capacity of multi-channel arbitrary networks is mainly limited by two factors: interference constraint and interface constraint [3] [11]. The capacity in arbitrary networks generally means transport capacity. The model in this paper is mainly different from the arbitrary network in the following point: the node in arbitrary network picks a destination node arbitrarily while the node in our model chooses the gateway node as its destination. As the gateway node becomes the maximum number of destination node in our model, the destination bottleneck constraint can be used to derive a tighter upper bound than interface constraint in arbitrary networks. Thus, the upper capacity bound in our model, as a special case of the arbitrary networks, is based on the interference constraint and destination bottleneck constraint.

4.1 Interference Constraint

The collision area can be characterized by interference area. Each node is equipped with multiple radio interfaces and each of them operates on different channels under the condition that channels used are non-overlapping. To simplify the capacity analysis, the switch overhead among
different channels is overlooked first. While a link $l$ is active between node $v$ and node $u$, all links transmitting to or receiving from this two nodes on the same channel should keep silent.

The per-node throughput highly depends on the number of channels that are transmitting simultaneously without interfering each other. Taken a specific channel into consideration, the links that are within the interference range with the same channel of this link is supposed to keep silent. Under the interference model, each hop of transmission is assumed to take up an area. Thus, the network throughput is remarkably affected by the maximum number of those areas within the range of whole coverage area.

**Theorem 1** The per-node throughput capacity of a ring-based many-to-one $(n, m, c)$ wireless mesh network is $O\left(\frac{W}{dL} \sqrt{\frac{m}{nc}}\right)$ bits/sec under protocol model.

**Proof:**

The proof is based on a proof in [3]. The nodes are assumed to be synchronized and the transmission time slot is notated as $T$. Since each node is assumed to produce $\lambda$ bits/sec. The node in $i$ ring is supposed to be $i$ hops away from the gateway node. Thus the average path length from each node to the gateway node is

$$L = \frac{\sum_{i=1}^{k} in_i}{n} = \frac{\sum_{i=1}^{k} (2i^2 - i)}{k^2} = \frac{(k + 1)(4k - 1)}{6k}.$$  \hspace{1cm} (5)

Consequently, the network capacity is can be defined as $\lambda nL$ bit-meters/sec.

Without loss of generality, we consider $t$ as any one second time interval. During time interval $t$, a bit $b$, $1 \leq b \leq n\lambda$, is assumed to be transmitted $h(b)$ hops on the path from its source node to the gateway node, where the $h$-th hop is $r_b^h$ in length. The distance traveled by a bit from its source node to the gateway node is at least equal to the length of the line joining the source and the destination (such as in canonical networks), thus we obtain:

$$\sum_{b=1}^{n\lambda} \sum_{h=1}^{h(b)} r_b^h \geq nL\lambda.$$ \hspace{1cm} (6)

The total number of hops traversed by all bits within $t$ is notated as

$$H = \sum_{b=1}^{n\lambda} h(b).$$

There are $m$ radio interfaces and each of them is assigned a specific channel. The channel can be switched. However, since the time interval is short enough, we can assume that the channel is predefined during this time interval and do not change over this time interval. Each channel has the capacity of $W/c$ bits/sec. Thus, there are maximum of $\frac{Wmn}{2c}$ bits that can be transmitted in one second by all nodes over all interfaces. That is

$$H \leq \frac{Wmn}{2c}. \hspace{1cm} (7)$$
According to the definition of protocol model, a transmission from one node to another with the length of $r$ is successful if and only if there is no simultaneous transmission within an area centered at the receiving node with the radius of $(1 + \delta)r$. When it comes to many-to-one network topology, all the traffic are directed to central gateway node. Thus, the placement of two simultaneous transmission can be considered as two cases, as shown in Fig. 5.

![Fig. 5: The placement of two simultaneous transmissions](image)

Case i: Both the transmission node and receiving node in two simultaneous transmission are located in the same straight line, such as the link $(v, u)$ and $(y, x)$ in Fig. 5. From the protocol model, we have

\[
\begin{align*}
d(y, u) &\geq (1 + \delta)d(v, u); \\
d(v, x) &\geq (1 + \delta)d(x, y),
\end{align*}
\]  

(8)

and

\[
\begin{align*}
d(y, u) &= d(x, y) + d(x, u); \\
d(v, x) &= d(v, u) + d(u, x).
\end{align*}
\]  

(9)

Combining Equation (8) and (9), we obtain

\[d(u, x) \geq \frac{\delta}{2}(d(x, y) + d(v, u)).\]

Even if all four nodes are located on the same side of gateway node, and suppose that link $(y, x)$ is closer to gateway node, we still have

\[d(u, x) \geq \frac{\delta}{2}(d(x, y) + d(v, u)) + d(x, y).\]

Case ii: Two concurrently transmitting links are located on different lines and both of them are directed to gateway node, as links $(v', u')$ and $(y', x')$ shown in Fig. 5. Using the protocol model and applying triangle inequality, we have

\[d(u', x') \geq \frac{\delta}{2}(d(x', y') + d(v', u')).\]
Considered the above two cases, the distance between receiving nodes of two links should be larger than \( \frac{\delta}{2} \) times the length of the hop around each receiver. The maximum distance between any two nodes of two simultaneous transmission links is defined as \( d_{\text{max}} \). By applying triangular inequality, we have

\[
d_{\text{max}} \leq d(x, y) + d(u, v) + d(x, u).
\]  

From \( d(x, u) \geq \frac{\delta}{2} (d(x, y) + d(v, u)) \) and the fact that \( \delta \geq 0 \) in protocol model, the minimum reuse distance \( d_{\text{reuse}} \) between two currently transmitting links can be obtained as

\[
d_{\text{reuse}} = \begin{cases} 
\delta(d(x, y) + d(u, v)) & \delta \geq 2; \\
(1 + \frac{\delta}{2})(d(x, y) + d(u, v)) & 0 \leq \delta < 2.
\end{cases}
\]

Based on the above analysis, the collision area is maximal when it is centered on a node, and the collision area is the area of a circle with radius \( d_{\text{reuse}}/2 \). In order to simplify the analysis, we assumed that \( \delta \geq 2 \) and \( d_{\text{reuse}} = \delta(d(x, y) + d(u, v)) \). Thus, each hop takes up a disk of radius \( \delta/2 \) times the length of the hop around each receiver. The area taken up by each channel is bounded above by the area of the domain (1 sqm). Taking all channels into account, we obtain

\[
\sum_{b=1}^{n} \sum_{h=1}^{h(b)} \pi \left( \frac{\delta}{2} \right)^2 (r_b^h)^2 \leq W,
\]

which can be rewritten as,

\[
\sum_{b=1}^{n} \sum_{h=1}^{h(b)} \frac{1}{H} (r_b^h)^2 \leq \frac{4W}{\pi \delta^2 H}.
\]  

(11)

As the expression on the left side is convex, we have,

\[
\sum_{b=1}^{n} \sum_{h=1}^{h(b)} r_b^h \leq \sqrt{\frac{4WH}{\pi \delta^2}}.
\]  

(12)

By substituting \( H \) from Equation (7), and combining with Equation (6) we obtain,

\[
\lambda \leq \frac{W}{\delta L} \sqrt{\frac{2m}{nc\pi}}.
\]  

(13)

When \( 0 \leq \delta < 2 \), the \( d_{\text{reuse}} = (1 + \frac{\delta}{2})(d(x, y) + d(u, v)) \). Each hop now takes up a disk of radius \( (1 + \frac{\delta}{2})/2 \) times the length of the hop around each receiver. By substituting this into the analysis presented above, we obtain

\[
\lambda \leq \frac{W}{(1 + \frac{\delta}{2})L} \sqrt{\frac{2m}{nc\pi}}.
\]  

(14)

Sum up the results of Equation (13) and (14), we have

\[
\begin{cases} 
\lambda \leq \frac{W}{\delta L} \sqrt{\frac{2m}{nc\pi}} & 0 \leq \delta < 2 \\
\lambda \leq \frac{W}{\frac{\delta L}{(1 + \frac{\delta}{2})L}} \sqrt{\frac{2m}{nc\pi}} & \delta \geq 2
\end{cases}
\]  

(15)
From the Equation (15) and Equation (5) we have, the per-node capacity under the interference constraint is $O\left(\frac{W}{\delta k} \sqrt{\frac{m}{nc}}\right)$ holding for $\delta \geq 0$.

### 4.2 Destination Bottleneck Constraint

As we known, most of traffic flows are directed to gateway node in many-to-one ring-based WMN, thus gateway node owns the maximum numbers of flows. Node $v'$, the closest node to gateway, is the bottleneck who determines the capacity [12]. The links that directly connected to the gateway node have the maximum collision load. In single channel single interface WMN, the bottleneck collision area is mainly located at the nodes that are directly connected with the gateway node. When it comes to multi-channels multi-interfaces WMN, the situation will be different. The bottleneck collision area might not be located at the nodes which are nearest to the gateway node. But, for the nearest nodes have the maximum traffic load, the links between them and gateway can be considered as the main clue to analyze the bottleneck collision area.

The collision area is closely related with the interference range and interference domain which we have analyzed in last subsection. As shown in Fig. 6, the collision area of the gateway node is the circle centered at the gateway node with the radius of $d_{reuse}/2$ where $r \leq d_{reuse}/2 < 2r$.

![Fig. 6: Interference range when a link exist between node $v$ and node $u$](image)

Let node $v$ be the node which is directly connected with gateway node with link $l(v)$ in one of channels with $\frac{W}{c}$ bits/sec and $T_i$ be the traffic load in each ring. The traffic load in the bottleneck collision area, notated as $T_{max}$, is

$$T_{max} = T_1 + \frac{\left(\frac{d_{reuse}}{2r}\right)^2 - r^2}{3r^2} T_2 = \sum_{i=2}^{k} n_i + \frac{\left(\frac{d_{reuse}}{2r}\right)^2 - r^2}{3r^2} \sum_{i=3}^{k} n_i.$$  \hspace{1cm} (16)

The gateway node is equipped with $m$ radio interfaces and a non-overlapping channel is assigned
to each interface. Thus, we have

\[ \lambda T_{\text{max}} \leq \frac{W_m}{c}. \]  

(17)

Substituting Equation (16) to (17), we obtain

\[ \lambda \leq \frac{W_m}{cT_{\text{max}}} = \frac{W_m}{cn} \frac{3(kr)^2}{(8r^2 + (d_{\text{reuse}})^2/4)(k^2 - 4)}. \]  

(18)

From Equation (18), the upper bound is determined when \( d_{\text{reuse}} \) reaches its maximum value. The value of \( d_{\text{max}} \), as defined in Equation (10), equals \( (2 + \delta)r \). Consequently, the per-node capacity determined by the bottleneck collision domain is as \( O\left(\frac{W_m}{cn\delta^2}\right) \).

Combining the analysis result under both interference constraint and bottleneck destination constraint, the per-node capacity in many-to-one multi-channel multi-radio wireless mesh networks is at most \( O\left(\min_o\left(\frac{W}{\delta k \sqrt{\frac{m}{cn}}, \frac{W_m}{cn\delta^2}}\right)\right) \) bits/sec. Based on this, the upper bound on per-node capacity of many-to-one WMNs is summarized as following theorem.

**Theorem 2** The upper bound on the throughput capacity of many-to-one wireless mesh networks under protocol model is as follows:

1. when \( \frac{c}{m} \) is \( \Omega\left(\frac{k^2}{m\delta^2}\right) \), the per-node capacity is \( \frac{W_m}{cn\delta^2} \) bits/sec.

2. when \( \frac{c}{m} \) is \( O\left(\frac{k^2}{m\delta^2}\right) \), the per-node capacity is \( \frac{W}{\delta k \sqrt{\frac{m}{cn}}} \) bits/sec.

5 Conclusion

The throughput capacity remarkably decreases as the number of mesh nodes increases. Therefore, the extent of capacity degradation needs to be considered during the designing period. In this paper, we firstly assumed that the many-to-one ring-based WMN is organized as a canonical network. Based on the analysis of bottleneck collision area and network connectivity, we obtained the upper capacity bound of \( O\left(\min_o\left(\frac{W}{\delta k \sqrt{\frac{m}{cn}}, \frac{W_m}{cn\delta^2}}\right)\right) \) bits/sec theoretically.

The result showed that the per-node throughput capacity of the many-to-one ring-based WMN is mainly determined by the ratio of the number of channels available and the number of radio interfaces. We also found that the number of rings and the guard zone parameter \( \delta \) have effect on the per-node capacity. This analysis result derived in canonical network model can be easily extended to general gateway node centered wireless mesh network model.

Besides, there are still lots of factors in throughput capacity analysis which we didn’t consider in this paper, e.g. channel assignment, radio interface assignment, fairness between one-hop nodes and multi-hop nodes, routing and traffic load and so on. We will address those in our future work.
References


