A Symbol-joint Resource Allocation for OFDM Systems with Asymptotical Fairness*

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Abstract

In general, the Marginal Adaptive (MA) algorithms are supposed to be fairness-oriented and the absolute fairness among users can be guaranteed in one symbol duration. In fact, the system performance is insensibly sacrificed to achieve such instantaneous fairness, which seems not very necessary or essential in practice for most applications. Instead, it is enough for users to get the same data rate after a period of a couple of symbols. This paper presents an asymptotically fair allocation algorithm named the Symbol-Joint Resource Allocation (J-SRA) based on the MA. It greatly exploits the time diversity by optimizing the subcarrier and power allocation for the successive $N$ symbols in time-frequency two-dimensional domain. The simulation results show that the proposed scheme achieve a better performance than the original one.

Keywords: Orthogonal Frequency Division Multiplexing (OFDM); Fairness; Resource Allocation; Symbol-joint Allocation

1 Introduction

The Orthogonal Frequency Division Multiplexing (OFDM) is very promising in supporting high-rate communications over hostile mobile environments and combating the Inter-symbol Interference (ISI) caused by multipath fading. It has become one of the key techniques of the fourth generation mobile communication system [1].

Performance and fairness are the two most important but contradictory parameters in optimization. It has been proved that to reach the best system performance, the resources should be assigned to the users with better channel status as much as possible; while if we want to keep the users equal data rates, more resources are required to allocated to the users with bad channel

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condition (as compensation), which may cause the system performance to decrease drastically. Normally, algorithms are categorized as Rate Adaptive (RA) [2, 3, 4, 5] and Marginal Adaptive (MA) [6, 7, 8, 9, 10] according to the different optimization objective. The objective of the RA is to maximize the capacity under constraints of transmit power and BER and the latter is to minimize the transmit power under the restrictions of capacity and BER.

In [2, 3], both the Max-Min and the SAE are fairness-oriented while algorithms in [4, 5] are capacity-oriented. In [11] Hamed tries to reach a compromise between capacity and fairness. For MA there seems little discussion about how to achieve a better balance between the system capacity and users’ fairness. It is because that the MA, innately, sacrifices the performance to maintain instantaneous fairness over every symbol [6, 7, 8, 9, 10]. But in fact for most applications and services, such accurate fairness is somewhat unnecessary. What the system needs to do is just to ensure that all the subscribers transmit equal amounts of data over a period of time—for example $N$ successive symbols—ignorant of their different instantaneous data rates.

Based on [9], a symbol-joint resource allocation (J-SRA) algorithm is proposed in this paper. Most of the existing MA algorithms are performing over one symbol duration and this is abandoned in J-SRA. Here we investigate and assign subcarrier, bit, power over $N$ successive symbols, which actually turns into a 2-dimensional optimization in time-frequency domain. System performance is considerably improved by making better use of the user diversity and time diversity.

The organization of this paper is as follows. In section II, we first give the system model and the definition of the fairness index. Besides, the goal of the resource allocation is also formulated. Section III describes the algorithm which includes initial resource allocation and iterative optimization. Section IV compares the performance between J-SAR and the original approach in [9] via Monte Carlo simulations. Finally, the paper is concluded in section V.

## 2 System Model and Problem Formulation

This section outlines the system model and states the allocation problem. The configuration of the multiuser adaptive OFDM system is shown in Fig. 1 [2].

Suppose that each subcarrier has a bandwidth that is much smaller than the coherence bandwidth of the channel. The channel status is assumed to be static in duration of a symbol but may change dramatically inter-symbols. Perfect Channel State Information (CSI) is assumed at both the receiver and the transmitter. $K$ users are involved in the system to share $N$ subcarriers. The $k$th user has a predetermined data rate of $R_k$ bits per OFDM symbol.

Let $r_n(k)$ denote the number of bits assigned to the $n$th subcarrier by user $k$. Furthermore, the transmission power allocated to the $n$th subcarrier of the $k$th user can be formulated as [9]

$$P_n(k) = \frac{f_k(r_n(k))}{G_{k,n}^2}$$

where the function $f(c)$ is the required received power with unity channel gain for reliable reception of $c$ bits and $G_{k,n}$ denotes the channel gain of subcarrier $n$. The total transmission power $P_T$ at
the source on all subcarriers and users for one symbol is expressed as

\[ P_T = \sum_{k=1}^{K} \sum_{n=1}^{N} P_n(k) \]  

(2)

Our objective is to minimize \( P_T \) under certain data rate requirement. Like most MA algorithms, the constraint in [8] is to keep all the consumers maintain the same rate on every symbol, i.e.

\[ R_k = \sum_{n=1}^{N} r_n(k) \]  

(3)

As described before, this constraint seems too stern which would restrain the transmission power from decreasing further. Therefore we relax it as

\[ \sum_{m=1}^{M} R_k^m = \sum_{m=1}^{M} \sum_{n=1}^{N} r_n^m(k) \]  

(4)

which indicates different instantaneous data rates are permitted but the same average rate over M consecutive symbols is kept for all of the user.

Besides the data rate, any subcarrier is required to be exclusively occupied by only one user at any given time. This monopolization can be written as: if \( r_n(k^*) \neq 0 \), then for any \( k \neq k^* \), \( r_n(k) = 0 \).

At last, we derive the optimization problem as follows:

\[ P_T^* = \min \frac{1}{M} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} P_n^m(k) = \min \frac{1}{M} \sum_{m=1}^{M} \sum_{k=1}^{K} \sum_{n=1}^{N} \frac{f_k(r_n^m(k))}{|G_{k,n}|^2} \]  

s.t.

\[ C1: \text{for } k \in \{1, 2, \cdots, K\}, \sum_{m=1}^{M} R_k^m = \sum_{m=1}^{M} \sum_{n=1}^{N} r_n^m(k) \]  

(5)

\[ C2: \text{for } n \in \{1, 2, \cdots, N\}, \text{if } \exists k^* \rightarrow r_n(k^*) \neq 0, \text{ then for } \forall k \neq k^* \rightarrow r_n(k) = 0 \]
Take note of that the first constraint is the data rate requirement and the second one ensures that one subcarrier can only be used by one user.

## 3 Subcarrier Bit and Power Allocation Algorithm

The Zhang algorithm in [9], combining the subcarrier allocation and the power allocation together, is constituted of two phases—the initial resource allocation and iterative optimization. In the first phase the Greedy is mainly used, which is famous for its high time consumption. Here we develop a fast allocation algorithm based on its channel state information in place of Greedy. The second phase is a two-dimensional optimization issue that enhances the performance by iterations.

### 3.1 Initial Resource Allocation

1) The water-filling Greedy algorithm [12]

The Greedy assigns data bit by bit, and at each time the subcarrier that requires the least additional power to transmit an additional bit is selected. It can be concluded as follows:

1. For all \( m \in \{1, 2, \ldots, M\} \), \( n \in \{1, 2, \ldots, N\} \), set \( r_{m,n}^m(k) = 0 \);  
2. Set \( \Delta p_n^m = \frac{f(r_{m,n}^m(k)+1)-f(r_{m,n}^m(k))}{G_{m,n}^2} \);  
3. select \((m^*, n^*) = \text{arg min} \Delta p_n^m\), update \( r_{n^*}^{m^*}(k) = r_{n^*}^{m^*}(k) + 1 \);  
4. Check \( \sum_{m=1}^M \sum_{n=1}^N r_{m,n}^m(k) < R_k \), if satisfied GOTO STEP 2);  
5. Finish.

2) The fast allocation algorithm

The water-filling principle mathematically reveals that the power assigned to certain subcarrier should have a positive correlation with the channel gain if the system wants to obtain its best performance. We propose a fast allocation scheme in terms of this theorem.

We define a factor CNR (Channel-gain-to-Noise Ratio) to indicate the quality of the channel, which is given by:

\[
\text{CNR}_i = G_i^2 / N_i
\]

where \( N_i \) is the noise power of the \( i \)th subcarrier. All users are allowed to use all the subcarriers as if this is a single-user OFDM system and we will deal with the channel collision in part 2. The bearing capability of subcarrier \( i \) is assessed to be proportional to the percentage that \( \text{CNR}_i \) accounts for of the overall subcarriers. Let coordinate \((m, n)\) marks the \( n \)th subcarrier in symbol \( m \) of user \( k \), and \( m, n \) will take value in the sets \( \{1, 2, \ldots, M\} \) \( \{1, 2, \ldots, N\} \) respectively. The fast allocation can be described as:

Performing the algorithm yields a matrix \( R \)—the rough scheme of initial subcarrier and bit allocation: \( R = \{r_{m,n}^m(k)|n = 1, 2, \ldots, N, m = 1, 2, \ldots, M, k = 1, 2, \ldots, K\} \).
3.2 Iterative Optimization

A serious problem occurred in part 1 is that one subcarrier may be taken up by more than one user, which is strictly forbidden as mentioned before. The main task of this part is to arbitrate the shared subcarriers—also called conflicting subcarriers in [9]—among all of the users. The iterative steps can be stated as follows:

1) If the result \( R \) derived from part 1 fulfills the constraint C2, it means no conflicting subcarrier occurred. Therefore the set \( R \) is the final allocation information and the whole algorithm comes to an end. Otherwise go to step 2).

2) Find all of the conflicting subcarriers and compute their conflicting power, which is defined to be the sum of the transmit power of users sharing this subcarrier \((m, n)\), given by:

\[
P_{mn} = \sum_{k=1}^{K} P_{mn}(k)
\]

3) Sort the conflicting subcarriers in a descending order of the conflicting power and the first subcarrier \((m^*, n^*) = \arg \max P_{mn}^*\) takes precedence. We try to reassign the bits on this subcarrier to other subcarriers for all colliding users and thus a new allocation scheme takes shape. Take user \( k \) for example, the data bits \( r_{mn}^*(k) \) loaded on subcarrier \((m^*, n^*)\) are reloaded to other ones and a new modulation matrix \( R'_k(k) = \{r'_{mn}(k) | n = 1, \cdots, N, m = 1, 2, \cdots, M\} \) for user \( k \) is derived. Recalculate the transmit power and the increase can be computed by:

\[
\Delta P_{mn}^*(k) = \sum_{m=1}^{M} \sum_{n=1}^{N} P(r'_{mn}(k)) - \sum_{m=1}^{M} \sum_{n=1}^{N} P(r_{mn}(k))
\]

4) Find the user \( k^* \) with the biggest transmit power increase, \( k^* = \arg \max_{1 \leq k \leq K} \Delta P_{mn}^*(k) \). Once the subcarrier \((m^*, n^*)\) is assigned to some user, the other colliding users have to reload the data on this subcarrier to other ones. As a result the overall transmitting power of the system must have an increase. In order to minimize the system power increase, the conflicting subcarrier \((m^*, n^*)\)
should be arbitrated to user $k^*$ because only in this way can the biggest user power increase be avoided. Therefore subcarrier and power allocation table of user $k^*$ remains $R(k^*)$ and the others’ will update to $\{R(k)|k \neq k^*\}$.

5) Delete the subcarrier $(m^*, n^*)$ from the conflicting subcarrier list and set the corresponding conflicting power $P_{m^*n^*}$ to zero. Repeat step 2, 3, 4 until the list is empty.

After the iteration hereinbefore, the ultimate subcarrier allocation and bit modulation information is derived. It can be used to modulate the information data directly.

### 3.3 Fairness Analysis

To evaluate the fairness of the proposed algorithm, a parameter fairness index [11] is introduced. The fairness can be defined in terms of several factors, such as power, data-rate or bandwidth. In this paper the data-rate is employed. Mathematically, it can be formulated as

$$f_m = \frac{\left(\sum_{k=1}^{K} \sum_{i=1}^{m} R_i^k\right)^2}{K \sum_{k=1}^{K} (\sum_{i=1}^{m} R_i^k)^2}, \quad m = 1, 2, \cdots, M$$

(9)

where $M$ is the maximum number of joint symbols.

When all of the users achieve the same rate, the fairness index reaches its maximum value 1 and the system is “extremely fair”. For the traditional MA algorithms, $f_m$ equals to 1 constantly. For the J-SRA, $f_m$ is a function of variable $m$ and converges to 1 asymptotically.

### 4 Simulation Results

In this section we present the performance of our proposed J-SRA scheme for multiuser OFDM systems by means of Monte Carlo simulations. The wireless environment is assumed to be six-path Rayleigh fading channels with M.1225 channel profile [13]. Additive white Gaussian Noise (AWGN) is present for all subcarriers and users, whose single-sided noise Power Spectral Density (PSD) level is $N_0$. Table 1 gives more details.

| Table 1: Simulation parameters |
|-------------------------------|-------------------|-----------------|
| Parameters  | Values  | Parameters  | Values  |
| Carrier frequency  | 3GHz  | Data rate  | 312bits/symbol/user |
| Channel bandwidth  | 5MHz  | BER  | $10^{-3}$ |
| FFT size  | 512  | PSD  | -103dBm |
| Subcarrier number  | 300  | Doppler frequency  | 5Hz/70Hz/300Hz |
| Sampling frequency  | 7.68MHz  | Paths  | 6 |

In simulations the M-ray Quadrature Amplitude Modulation (MQAM) is employed with $M = \{0, 2, 4, 6\}$. We use the Square signal constellations to carry 2, 4, or 6 bits/symbol. The required power for transmitting $c$ bits at certain Bit Error Rate (BER) $P_e$ is [14]

$$f(c) = \frac{N_0}{3} \cdot \left[ Q^{-1} \left( \frac{P_e}{4} \right) \right]^2 \cdot (2^c - 1)$$

(10)
where

\[ Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^\infty e^{-\frac{t^2}{2}} dt \]  

(11)

Fig. 2 compares the transmitting power of Zhang in [9] and J-SRAs. The digit following J-SRA denotes the joint size of the subcarriers, i.e. the value of the parameter M. Indoor office environment with 5 Hz Doppler frequency shift is assumed. Firstly and basically, the simulation result shows that J-SRA outperforms Zhang algorithm in [9]. Secondly, the average power per symbol increases with the number of the users as more users have more data to be transmitted.

Fig. 3 shows fairness of a multiuser OFDM system with 2, 4, 6 and 8 users respectively. The joint size M equals 7. It can be seen that all curves converges to 1 asymptotically but at different speeds. The curve of 2 users goes very smoothly and is close to 1 all along while the curve
of 8 users starts at a low point and climbs up drastically to 1. Fig. 2 and Fig. 3 verifies the contradiction between system performance and subscriber fairness. Fairness among users can be sacrificed to an acceptable extent to achieve better performance in turn.

Fig. 6 compares the fairness among different wireless environments. To some degree, J-SRA prefers rough circumstances with large Doppler frequency shift.

![Fig. 6: Comparison of fairness with Doppler shift equals 5/70/300 Hz, M = 7](image)

5 Conclusion

In this paper a symbol-joint subcarrier, bit and power allocation scheme is proposed. It contains two phrases: in phrase 1 initial resource assignment is performed regardless of collisions among the users, and in phrase 2, arbitrate the conflicting subcarriers over consecutive M symbols. Simulation results show that the proposed approach outperforms Zhang algorithm in [9] by exploiting the multiuser diversity and time diversity more effectively and efficiently. Instead of the instantaneous fairness, the fairness converges asymptotically to 1 which is acceptable in most transactions and services.

References

[1] 3GPP TS 36.201 V8.3.0 LTE Physical Layer-General Description (Release 8), 09, 2009, 6-9


