Wavelet-neural-network Based Robust Sliding-mode Control for Induction Motor *

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Abstract

This paper presented a speed estimation and control strategy for induction motor drive based on an indirect field-oriented control. The rotor speed estimator based on a wavelet-neural-network utilized stator voltage and current measured values to calculate the rotor speed, and the control approach based on a sliding-mode controller with an integral sliding surface was proposed in order to regulate the induction motor speed. The stability analysis of the proposed control approach under parameter variations and load disturbances was provided using the Lyapunov’s stability arguments. Simulation results show that the presented scheme provides high-performance dynamic characteristics and that this approach is robust with respect to plant parameter uncertainties and external load disturbances.

Keywords: Wavelet Neural Network; Sliding-mode Control; Robust Control; Induction Motor Drive

1 Introduction

As one of the most popular techniques, the Field-oriented Control (FOC) technique has been widely used in industry for high-performance Induction Motor (IM) drive [1], where the knowledge of synchronous angular velocity is often necessary in the phase transformation for achieving the favorable decoupling control. However, the performance is sensitive to the variations of motor parameters, which varies with the temperature and the saturation of the magnetizing inductance. Moreover, the control performance of the IM is seriously influenced by the uncertainties, such as mechanical parameter variation, external disturbance, unstructured uncertainty due to non-ideal field orientation in the transient state, and un-modeled dynamics. However, it is usually very difficult to get the complete information of uncertainties.

In the past decade, active research has been carried out on neural networks in the control field. Especially, much research has been done on applications of Wavelet Neural Networks (WNNs), which combine the capability of artificial NNs for learning from processes and the capability of...
wavelet decomposition \cite{2,3} for identification and control of dynamic systems \cite{4,5,6,7}. Seshagiri et al. \cite{8,9} described a wavelet-based NN for function learning and estimation, and the structure of this network was similar to that of the Radial Basis Function (RBF) network except that the radial functions were replaced by orthogonal scaling functions. From the point of view of function representation, the traditional RBF networks can represent any function that is in the space spanned by the family of basis functions. However, the basis functions in the family are generally not orthogonal and are redundant. It means that the RBF network representation for a given function is not unique and is probably not the most efficient. In this study, the family of basis functions for the RBF network is replaced by an orthogonal basis (i.e., the scaling functions in the theory of wavelets) to form a WNN \cite{9}. To overcome the above errors, it is possible to design the estimators of the machine quantities using WNNs, which do not require a mathematical model of the drive system and, therefore, the performance of this approach does not exhibit any dependence on the modeling errors.

In FOC control scheme, the control performances of the induction motor system are always influenced by the uncertainties, which usually are composed of unpredictable parameter variations, external load disturbances. As an effective and robust control approach, Sliding-Mode Control (SMC) is widely used to deal with nonlinear systems with both system parameter variations and external disturbances. The essential property of SMC is that the discontinuous feedback control switches on one or more manifolds in the state space. The SMC can offer good properties, such as insensitivity to parameter variations, external disturbance rejection, and fast dynamics response. Upon these advantages so far, many research have been reported for IM drives control or estimation, using sliding-mode technique \cite{10,11,12}.

This paper presents a sensorless vector control scheme consisting of a WNN based speed estimation algorithm and of a SMC controller. The estimation based on a WNN utilizes stator voltage and current measured values to calculate the speed. Unlike the traditional variable structure designs, a SMC controller with an integral sliding surface is proposed here in order to regulate the motor speed. Using this SMC controller, the controlled speed is insensitive to the uncertainties in the motor parameters and load disturbances, and besides the acceleration signal used in conventional variable structure speed control is not required.

2 WNN Based Speed Estimation of Induction Motor

2.1 Mathematical Model of Induction Motor

A WNN will be designed to estimate the rotor speed. Various input variables to the NN can be considered such as stator voltages and currents, stator and rotor fluxes, etc. The rotor voltage equations of a squirrel cage induction motor drive in the stationary frame may be written as \cite{1}

\[
\begin{align*}
    u_{ra} &= R_r i_{ra} + L_r \dot{i}_{ra} + L_m \dot{i}_{sa} + \omega_r L_m i_{s\beta} + \omega_r L_r i_{s\beta} \\
    u_{rb} &= R_r i_{rb} + L_r \dot{i}_{rb} + L_m \dot{i}_{sb} - \omega_r L_m i_{s\alpha} + \omega_r L_r i_{s\alpha}
\end{align*}
\]

where $u$ is the voltage, $L$ is the inductance, $R$ is the resistance, $i$ is the current, and $\omega_r$ is the rotor electrical speed. The subscript $r$ denotes the rotor values, the subscript $s$ denotes the stator values, and the subscripts $\alpha$ and $\beta$ denote the $\alpha\beta$-axis components in the stationary reference
frame. In the same way, the stator voltage equations in the stationary frame may be written as

\[
\begin{align*}
    u_r^\alpha &= R_s i_s^\alpha + L_s \dot{i}_s^\alpha + L_m \dot{i}_r^\alpha \\
    u_r^\beta &= R_s i_s^\beta + L_s \dot{i}_s^\beta + L_m \dot{i}_r^\beta
\end{align*}
\]

(2)

By re-arranging of Eq. (1), the rotor speed can be derived:

\[
\omega_r = \frac{L_m \dot{i}_r^\alpha + R_r i_r^\alpha + L_r \dot{i}_r^\alpha}{L_m i_s^\beta + \omega_r L_r i_s^\beta}
\]

(3)

Eq. (3) reveals that the rotor speed can be found through calculations of stator and rotor currents. From Eqs. (1) and (2), the rotor derivatives with respect to time can be found in terms of stator voltages and currents that are easy to measure:

\[
\begin{align*}
    \dot{i}_r^\alpha &= \frac{u_s^\alpha}{L_m} - \frac{R_s i_s^\alpha}{L_m} - \frac{L_s \dot{i}_s^\alpha}{L_m} \\
    \dot{i}_r^\beta &= \frac{u_s^\beta}{L_m} - \frac{R_s i_s^\beta}{L_m} - \frac{L_s \dot{i}_s^\beta}{L_m}
\end{align*}
\]

(4)

Integrating the previous equations, we obtain

\[
\begin{align*}
    i_r^\alpha &= \frac{1}{L_m} \int \left( \frac{u_s^\alpha - R_s i_s^\alpha}{L_m} \right) dt - \frac{L_s \dot{i}_s^\alpha}{L_m} \\
    i_r^\beta &= \frac{1}{L_m} \int \left( \frac{u_s^\beta - R_s i_s^\beta}{L_m} \right) dt - \frac{L_s \dot{i}_s^\beta}{L_m}
\end{align*}
\]

(5)

Substituting Eqs. (4) (5) in Eq. (3), relationships between speed and stator variables (voltages and currents) in the stationary reference frame are concluded as a function mapping:

\[
\omega_r = f(u_s^\alpha, u_s^\beta, i_s^\alpha, i_s^\beta)
\]

(6)

where \( f() \) represents a nonlinear function. Therefore, we can conclude that the rotor speed is a nonlinear function of the stator voltage and current, and that these variables will be an adequate input signals to estimate the rotor speed using a NN. The WNN has four input signals, the stator voltages and currents, and one output \( \hat{\omega}_r \), the estimated rotor speed.

2.2 WNN Based Speed Estimation

WNN was independently proposed by many researchers [5, 6, 7, 8] and are a popular alternative to the MLP. The objective of this article is to adopt WNN to approximate the speed. The available learning vectors should be used to determine the number of radial basis neurones and their governing parameters in an unsupervised way. The input and output vector of WNN is \( X = [u_s^\alpha, u_s^\beta, i_s^\alpha, i_s^\beta]^T \). NNs output \( Y = \hat{\omega}_r \) gives the estimated speed. The WNN in this paper is feed-forward network with a hidden layer. The weights of input layer are fixed, and the active function adopts Mexican hat function with the output \( Y = \sum_{j=1}^J w_j (\sum_{i=1}^I m_{ij} x_i - d_j) \), where \( I, J \) are the node number of input hidden layers, respectively; \( \varphi \) is the wavelet basis function; \( m_{ij}, d_j, w_j \) are the scaling factor, delaying factor and connected weight of \( j \)th hidden node. The network learning includes two phases: structure learning and parameters learning.
(1) Structure Learning. This phase is used to construct a proper network from the training set with an $m$ labeled pair $X_j, d_j$ and to minimize the number of neurons in the network. The hidden units are initialized with random centers and widths for each run and added one by one according to the so-called novelty condition. The first step to determine whether or not to perform the structure learning. If $e_{min} \leq |e_m|$, where $e_m$ is preset positive constant, then the structure learning is necessary. Next, it will further decide whether or not to add a new neuron in the hidden layer by using the novelty measure. The error between the sampled and desired values is obtained and used as the novelty measure as $e(i) = Z(i) - Y(i)$, $d(j) = ||X(i) - C_j||$ for $j = 1, 2, \ldots, m$, where $Z(i)$ is the desired output, $Y(i)$ is sampled output, $m$ is the number of exit units in the hidden layer and $d(j)$ denotes the Euclidean distance between the exit units of the hidden layer and the $i$th sample. According to the novelty measure, the criterion for generating a new WNN neuron for new incoming data is described as follows.

Firstly find the minimal Euclidean distance $d_{min} = min(d_j)$. If $||e(i)|| \geq \epsilon$ and $d_{min} \geq \eta(i)$, then the novelty condition is satisfied and a new neuron is inserted into the hidden layer, where $\epsilon$ is the desired precision, $\eta(i)$ is the fitting accuracy of the WNN and decreases from $\eta_{max}$ to $\eta_{min}$, $\gamma$ is the attenuation factor whose value is in the range [0, 1]. Next, the initial parameters of the new neuron are assigned with preset values as $m_{ij} = X(i), d_j = 0.5(\sum_{q=1}^{2} ||X(i)|| - C_q)^{0.5}$, where $C_q$ is the centre of the two units of hidden layer with nears Euclidean distances to input sample $X(i)$. To obtain a simply and compact architecture, a deletion strategy is implemented to cutoff some certain units, which are not active any more and contribute little to the NN’s output. If the ratio of weighted output to actual output for the successive $n$ input samples satisfies the following inequality, $W_jZ_j/Y(i) \leq \delta$, where $\delta$ is a pre-specified threshold value and $n$ is a positive number. Then the deletion strategy is done and the $j$th unit must be cutoff from the hidden layer. Once the number of basis functions used, their centers and their widths are determined, recursive least squares algorithm is adopted to train the connected weights. After the unsupervised training, the architecture of the proposed WNN is determined as well as the network parameters are initialized.

(2) Parameter Learning. Parameter learning is used to adjust the parameters based on supervised learning algorithms to adjust the the forward weights and the parameters of WNN neurons using the gradient descent algorithm to minimize a given energy function [6]. The energy function is firstly defined as $E(n) = 0.5 \sum_{i=n-M}^{n} e^2(i)$, where $n$ is the number of input samples, and $M$ is the update window size and equals the number of time instants over which the gradient of the cost function $E$ is computed.

The update of the scaling factors and delaying factors of the hidden layer is

$$\begin{align*}
\begin{cases}
m_{ij}(t+1) = m_{ij}(t) - \eta \frac{\partial E}{\partial m_{ij}} + \alpha \Delta m_{ij}(t-1) \\
d_{j}(t+1) = d_{j}(t) - \eta \frac{\partial E}{\partial d_{j}} + \alpha \Delta d_{j}(t-1)
\end{cases}
\end{align*}$$

where $\eta$ are the learning-rate parameters of the WNN.

The updating laws of the connected weight vector $w_{jl}$ between the hidden and output layer are

$$\begin{align*}
\begin{cases}
\mathbf{w}(t) = \mathbf{w}(t-1) + \mathbf{K}(t)[y(t) - \varphi^T(t)\mathbf{w}(t-1)] \\
\mathbf{K}(t) = [\lambda + \varphi^T(t)\mathbf{P}(t-1)\varphi(t)]^{-1} \mathbf{P}(t-1)\varphi(t) \\
\mathbf{P}(t) = \lambda^{-1}[\mathbf{P}(t-1) - \mathbf{K}\varphi(t)\mathbf{P}(t-1)]
\end{cases}
\end{align*}$$

where $0 < \lambda \leq 1$ is the forgetting factor, $\varphi(t) = [\varphi_1, \varphi_2, \ldots, \varphi_J]^T$ is the output and weight vector at time $t$, $\mathbf{I}$ is unit matrix, and $\gamma > 0$. 

3 Sliding-mode Speed Control of IM System

As it is well known, the mechanical equation of an induction motor can be written as

\[ J\dot{\omega}_m + B\omega_m + T_L = T_e \]  \hspace{1cm} (9)

where \( J \) and \( B \) are the inertia constant and the viscous friction coefficient of the induction motor system, respectively; \( T_L \) is the external load; \( \omega_m \) is the rotor mechanical speed in terms of angular frequency, which is related to the rotor electrical speed by \( \omega_m = 2\omega_r/p \) where \( p \) is the pole numbers and \( T_e \) denotes the electromagnetic torque of an induction motor, defined as \( T_e = \frac{3pL_m}{4L_r}(\varphi_{dr}i_{qs} - \varphi_{qr}i_{ds}) \), where \( \varphi_{dr} \) and \( \varphi_{qr} \) are the rotor-flux linkages, \( i_{ds} \) and \( i_{qs} \) are the stator currents in the synchronously rotating reference frame.

Using the field-orientation control principle, the current component \( i_{ds} \) is aligned in the direction of the rotor flux vector \( \varphi_r \), and the current component \( i_{qs} \) is aligned in the direction perpendicular to it. With this condition, it follows that \( \varphi_{qr} = 0 \) and \( \varphi_{dr} = |\varphi_r| \). The slip frequency is \( \omega_{sl} = \omega_e - \omega_r \) and can be calculated as \( \omega_{sl} = \frac{L_m}{L_r}\varphi_{qs} \), where \( \omega_e \) is the stator supply frequency, and \( T_r = L_r/R_r \) is the rotor time constant. Taking into account the results of field-oriented vector control, the equation of induction motor torque is simplified to \( T_e = \frac{3pL_m}{4L_r}\varphi_{dr}i_{qs} \), where \( K_T \) is the torque constant.

Then, the mechanical equation (9) becomes \( \dot{\omega}_m + a\omega_m + f = bi_{qs} \), where the parameters are defined as \( a = BJ^{-1} \), \( B = K_TJ^{-1} \), \( F = T_LJ^{-1} \). Consider the previous mechanical equation with uncertainties as follows

\[ \dot{\omega}_m = -(a + \Delta a)\omega_m - (f + \Delta f) + (b + \Delta b)i_{qs} \]  \hspace{1cm} (10)

where the terms \( \Delta a \), \( \Delta b \) and \( \Delta f \) represent the uncertainties of the terms \( a \), \( b \) and \( f \), respectively.

Define the tracking speed error as \( e(t) = \dot{\omega}_m(t) - \omega_m^*(t) \), where \( \omega_m^*(t) \) is the rotor speed command, and \( \dot{\omega}_m(t) \) is the estimated rotor speed. Considering the derivative of the previous equation with respect to time yields \( \dot{e}(t) = -ae(t) + u(t) + d(t) \), where the signal \( u(t) = bi_{qs} - a\omega_m^* - f - \dot{\omega}_m^* \) has collected all the related terms and the uncertainty terms have been collected as \( d(t) = -\Delta a\omega_m - \Delta f + \Delta bi_{qs} \).

Now, the sliding variable \( S(t) \) with an integral component can be defined as

\[ S(t) = e(t) - \int_0^t (K - a)e(\tau)d\tau \]  \hspace{1cm} (11)

where \( K \) is a constant gain and must be chosen so that the term \( (K - a) \) is strictly negative. Then, the sliding surface is defined as \( S(t) = 0 \).

The sliding-mode controller is designed as

\[ u(t) = Ke(t) - \beta sgn(S) \]  \hspace{1cm} (12)

where \( sgn() \) is the sign function, and \( \beta \) is the switching gain and must be chosen so that \( \beta > |d(t)| \) holds for all time.

If the gain \( K \) and are properly chosen, the control law (12) can lead to the desired rotor mechanical speed and the speed tracking error tends to zero as the time tends to infinity. The proof of this theorem will be carried out using the Lyapunov’s stability arguments. Define the Lyapunov
function $V(t) = 0.55^2(t)$. By using Eqs. (10), (11) and (12) and the inequality $\beta > |d(t)|$, its time derivative is calculated as

$$\dot{V}(t) = S(t)\dot{S}(t) = S(t)[\dot{e} - (K - a)e] = S(t)[d(t) - \beta \text{sgn}(S)]$$

(13)

Using the Lyapunov’s direct method, since $V(t)$ is clearly positive definite, $\dot{V}(t) \leq -(\beta - |d|)|S| \leq 0$ is negative definite, and $V(t)$ tends to infinity as $S(t)$ tends to infinity. Therefore, $S(t)$ tends to zero as the time $t$ tends to infinity, then $S(t) = \dot{S}(t) = 0$, and the dynamic behavior of the tracking problem and the tracking error $e(t)$ converges to zero exponentially. Finally, the torque current command $i_{qs}^*(t)$ can be obtained as

$$i_{qs}^*(t) = b^{-1}[K e(t) + a\omega_m^* + f(t) - \beta \text{sgn}(S)]$$

(14)

Therefore, the proposed variable structure speed control resolves the speed tracking problem for the induction motor with some uncertainties in mechanical parameters and load torque.

4 Simulation Results

In our MATLAB/SIMULINK based simulation, we verify our control scheme discussed in Section 3. Fig. 1 presents the block diagram of the proposed robust control scheme. The block ‘WNN’ is the neural network designed in Section 2 for estimating the rotor speed. The induction motor used in this case study is a 50HP, 460V, 2 pole, 60 Hz motor having the following parameters: $R_s = 0.187 \Omega$, $R_r = 0.15 \Omega$, $L_s = 69.9\text{mH}$, $L_r = 69.9\text{mH}$, $L_m = 68\text{mH}$, $J = 0.0586 \text{kg} \cdot \text{m}^2$ and $B = 0.1 \text{Nm} \cdot \text{s}$. It is assumed that there is an uncertainty of around 250% in the system parameters, which will be overcome by the proposed sliding control.

Fig. 2 gives the training data used in this work. When simulating, these data will help the controller construct the WNN. After about 6000 iterative echoes, the estimation error is less than $1 \times 10^{-4}$, and the desired accuracy is obtained for further estimation.

Fig. 1: Block diagram of the proposed WNN based sliding-mode robust control
Fig. 2: Training data of the rotor speed, stator voltage and stator current

Fig. 3 gives the speed estimation result and its error under no load. At this case, the motor starts from a standstill state and we want the rotor speed to follow a speed command that starts from zero and accelerates until the rotor speed is 125 rad/s. At time $t = 0.5s$, the rotor speed accelerates again until the rotor speed is 150 rad/s. The motor starts with no load, and at time $t = 2s$ the load torque steps from $T_L = 0$ to 40 Nm. Moreover, the estimator model of IM presents higher uncertainties of around 300% in $R_s$ and 250% in $R_r$ at time $t = 3s$.

Fig. 4 shows the desired rotor speed the tracking error between the desired rotor speed and the real rotor speed. As it may be observed, the error is close to zero at most of the simulation period and the rotor speed tracks the desired speed in spite of system uncertainties. Moreover,
the speed tracking is not affected by the load torque change at time $t = 2s$, because when the sliding surface is reached, the system becomes insensitive to the boundary external disturbances.

5 Conclusion

This paper presents a sliding-mode vector control with WNN rotor speed estimation. The WNN employed is composed of a feed-forward multi-layer NN. The weights and biases of the NN are updated using a gradient descent training algorithm. The inputs to the NN are the measured stator voltages and currents, and the output of the NN is the estimated rotor speed. In addition, a variable structure control is proposed, which has an integral sliding surface in order to relax the requirement of the acceleration signal, as is usual in conventional sliding mode speed control techniques. The closed-loop stability of the presented design has been proved through Lyapunov stability argument. Simulation results have shown that the proposed control scheme performs well and that the speed tracking objective is achieved in spite of uncertainties both in the system parameters and in the load torque.

References