The Modeling and Simulation of Supply Chain Based on Directed Complex Network

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Abstract

In recent years, the study of directed complex network is becoming a hot issue. Complex networks describe a wide range of systems. In this paper, based on the complex network theory, we find that supply chain network is clearly not a random network but a directed scale-free network. Statistic characteristics of the network model such as degree distribution, supply chain average shortest path and clustering in four different situations are studied. The result has showed the rule of supply chain, its power-law exponent of degree, clustering and average shortest path.

Keywords: Complex Network; Supply Chain; Directed; Statistic Characteristics; Modeling and Simulation

1 Introduction

Complex networks system play an important role in real life. In recent years, the study of directed complex network is becoming a hot issue. Many researchers begin to study the directed complex network model [1-3] and analyze its statistical characteristics such as degree and clustering [4-7], etc. Commonly cited examples include telephone call network [8], social network [9], communication network [10] and World-Wide Web [11], etc. Complex networks can describe the patterns and features of the real word [12]. To model such a local-world effect, a local-world evolving network model is proposed. However, it doesn’t often describe and analyze the evolution process and the whole performance of supply chain.

Supply chain can be described as a complex network structure with various business entities and business relation. Through the operation of supply chain, we can find that: (1) Every business entity of supply chain has its special property, including suppliers, sellers and customers. (2) Every business entity of supply chain has its special location or order. That means the relation of connected business entity has direction, which from upstream business entity to downstream business entity. (3) Supply chain is dynamically growing.

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Motivated by these observations, we propose a directed complex network model for supply chain. The business entities of supply chain are divided into three categories, suppliers, sellers and customers. The entities of different category have different business scale and scope.

The rest of this paper is organized as follows. In Section 2, we designed the directed supply chain complex network. In Section 3, we analyze statistical characteristics of supply chain. The modeling and simulations on supply chain complex network properties in 4 situations are showed in Section 4. Finally, the main conclusion is made in Section 5.

2 The Definition of the Directed Supply Chain Complex Network

A directed complex network is a collection of individuals and communities. In this paper we assume the directed supply chain complex network to be represented as a graph $G = (V, E)$ with a set $V$ of $N$ node business entities and a set $E$ of $M$ directed edges $(v_i, v_j)$. Distinguishing with undirected network, it has specific destination. As it to say, wherever flux flow, it is always flowing from node $i$ and finally arriving node $j$. From Fig. 1, we can find the difference between the two. There is the undirected supply complex network in Fig. 1 (a), and Fig. 2 (b) manifests the directed one.

We assume each business entity is represented by a node which defined as $v_i = (ID, Category, Level)$, where $ID$ is the number of node, $Category$ is the label of node property, $Level$ is showed the business order in supply chain network.

The edge of supply chain network represent the operational relations between node $i$ and $j$. Edge is denoted as $e = (v_i, v_j)$, said $e$ is the associated edge. $v_i$ is the starting point of the edge, $v_j$ is called the end point of edge. The level of the $v_i$ must be smaller than $v_j$ to ensure that the edge direction is the same as supply chain logistics direction. Because of directed supply chain network, the edge from node $i$ to node $j$ is different from the edge from $j$ to $i$, respectively $i \to j$ and $j \to i$. 
3 Statistical Characteristics for the Parameters of Networks

3.1 Degree

The simplest and the most intensively studied node characteristic is degree. For supply chain, degree symbolizes the synthesis operation ability of an entity. Degree (or connectivity), \( k(i) \), of a node \( i \) is the total number of its connections. In-degree, \( k_{in} \), is the number of incoming edges of a node. Out-degree, \( k_{out} \), is the number of outgoing edges of a node. Hence, \( k = k_{in} + k_{out} \). Degree is actually the number of nearest neighbors a node. In-degree means the supply channel and out-degree means the sale channel.

The distribution of degrees in a network can be described by the distribution function \( P(k) \). \( P(k) \) is the probability that a node has a total of \( k \) edges which connect to other nodes, \( P(k)_{in} \) is the probability that a node has \( k \) incoming edges directed to itself (probability of supply chain from \( k \) supply channel), \( P(k)_{out} \) is the probability that a node has a total of \( k \) edges to other nodes (probability of supply chain to \( k \) sale channel). Most networks have degree distributions, which have a power law tail \( P(k) \sim k^{-\theta} \).

3.2 Average Shortest Path

The average path length \( L \) of the network represents distance between two nodes, averaged over all pairs of node. When a node is added, each node of the network according to the time is marked. The average path length \( L \) of a network is defined to be the average value of all distances over the network:

\[
L = \frac{1}{N(N-1)/2} \sum_{i \geq j} d_{ij}.
\]

Here, \( d_{ij} \) is the average shortest path length from node \( i \) to \( j \), \( N \) is size of the network, i.e., the total number of nodes in the network.

From the above formula, we can find that \( L \) decreases as \( N \) increases. In supply chain network, a supply chain path should start from lowest-level supplier (\( k_{in} = 0 \)) to customer. So the supply chain path length \( d_{ij} \) is the number of links between a lowest-level supplier node and a customer node, the supply chain average path length \( L \) is the average number of links between supplier node and customer node.

3.3 Clustering Coefficient

In supply chain network, it is quite possible that your supplier’s supplier is also your directed supplier, or, to put it another way, two of your suppliers are quite possible supplier of each other. This phenomenon is characterized by the concept of clustering.

Clustering coefficient \( (C_i) \) of a node is defined as the ratio of number of links shared by its neighboring nodes to the maximum number of possible links among them.

\[
C_i = \frac{2E}{k_i(k_i - 1)}
\]

where \( E \) is the number of edges among the neighboring set of node \( i \).
In other words, \( C_i \) is the probability that two nodes are linked to each other given that they are both connected to node \( i \). The average clustering coefficient \( C \) is defined as the average clustering coefficient of each node.

\[
C = \frac{1}{N} \sum_i C_i
\]

Clustering coefficient captures the local cohesiveness of a node. Average clustering coefficient \( C \) measures the global density of interconnected nodes in the network.

4 Modeling and Simulation

In supply chain network, we assume the supply chain has 2560 nodes and initial parameters are set as follows:

(i) \( m_0=30 \), \( m_0 \) is the number of the nodes in the network before no growth.
(ii) \( m_1=3 \), \( m_1 \) is the number of edge of new supplier which enter into the network.
(iii) \( m_2=5 \), \( m_2 \) is the number of edge of new seller which enter into the network.
(iv) \( m_3=3 \), \( m_3 \) is the number of edge of new customer which enter into the network.
(v) the level of suppliers is a random number from 1 to 30, the sellers is from 31 to 200 and the customer is 201.

We perform the simulation with the initial parameters and set up 4 situation: situation 1: \( N_1:N_2:N_3=5:5:5 \), situation 2: \( N_1:N_2:N_3=4:7:4 \), situation 3: \( N_1:N_2:N_3=4:4:7 \), situation 4: \( N_1:N_2:N_3=7:4:4 \), where \( N_1 \) is the number of suppliers, \( N_2 \) is the number of sellers, \( N_3 \) is the number of customers. Each experiment is performed 50 times. The average is used to analysis the network character.

4.1 Node Degree Distribution

The degree distributions decay is greatly faster than the power-law degree distribution. From Fig. 2, it can be found some regular results that the node degree of directed supply chain network obey power-law distribution. The power-law exponent of all nodes degree distribution = \(-2.9 \pm 0.1\). The power-law exponent of degree distribution of all suppliers=\(-3.5 \pm 0.05\). The power-law exponent of degree distribution of all sellers=\(-2.6 \pm 0.05\). The power-law exponent of degree distribution of all customer=\(-4.7 \pm 0.05\).

That means the most business entities have relatively less business relation with other entities, but few have a large number of business relation with other entities and become the core of supply chain. Compare the degree distribution of different category of nodes, we can find the degree distribution of sellers is more well-proportioned and the customer not.

At the beginning of supply chain growth, the scale of supply chain is small. As the increase of business scale, some enterprises just joined in the supply chain are going to build business relation with old business entities. In supply chain, the business entities with good reputation, credit and operational capacity will have more opportunity to build business relation with new added enterprise. So the structure of supply chain will change because of the new added enterprise, and some business entities will grow faster finally to become the core enterprise of supply chain.
Fig. 2: The degree distribution of directed supply chain network

Fig. 3 is the double logarithmic distribution graphs of directed supply chain network node in-degree and out-degree.

An exponential network is provided by the usual random graph, with $P(k)$ decreasing exponentially fast. In our case, the in-degree and out-degree distributions decrease exponentially fast, cumulative distribution functions.

4.2 Average Shortest Path

From Table 1, we can find the average supply chain path length is nearly to 2.86 with scale of supply chain increasing. We can say that there is 2.86 degrees of separation between supply chain
business entities, or, in other words, the supply chain network is a small-world.

<table>
<thead>
<tr>
<th>Experiment number</th>
<th>Supplier</th>
<th>Seller</th>
<th>Customer</th>
<th>Supply chain average shortest path</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>120</td>
<td>200</td>
<td>320</td>
<td>2.4944</td>
</tr>
<tr>
<td>2</td>
<td>240</td>
<td>400</td>
<td>640</td>
<td>2.7175</td>
</tr>
<tr>
<td>3</td>
<td>360</td>
<td>600</td>
<td>960</td>
<td>2.8737</td>
</tr>
<tr>
<td>4</td>
<td>480</td>
<td>800</td>
<td>1280</td>
<td>2.8537</td>
</tr>
</tbody>
</table>

### 4.3 Clustering Coefficients

With the scale of supply chain increased, the Average clustering coefficient C is nearly to 0.03. The result show the supply chain has a low average clustering coefficient C which manifest Table 2.

<table>
<thead>
<tr>
<th>Experiment number</th>
<th>Supply chain average clustering coefficient</th>
</tr>
</thead>
<tbody>
<tr>
<td>600</td>
<td>0.04375</td>
</tr>
<tr>
<td>1200</td>
<td>0.04215</td>
</tr>
<tr>
<td>1800</td>
<td>0.03550</td>
</tr>
<tr>
<td>2400</td>
<td>0.02897</td>
</tr>
<tr>
<td>3000</td>
<td>0.02500</td>
</tr>
</tbody>
</table>

![Graph](image.png)  # Fig. 4: The relation of average clustering coefficient and degree
Additional information at the intermediate level can be obtained by defining $C(k)$, average clustering coefficient of nodes with degree $k$. Fig. 4 show the $C(k) \sim k$.

It has been shown that supply chain network is expected to have a non-trivial, power-law decay of $C(k)$ as a function of $k$. $C(k)$ shows a power-law decay with $C(k) \sim k^{-0.745}$. Many real networks have been found to have such nontrivial decay.

5 Conclusions

This paper provides a entirely new perspective to the study of supply chain. The basic character of supply chain is complexity, dynamic and growth. In this paper, we have studied the statistic characteristics of supply chain network model. The degree symbolizes the synthesis operation ability of an entity. The supply chain average path length is the average number of links between a supplier node and a customer node. The average clustering coefficient basically shows the global density of interconnected nodes. In 4 different situations, degree distribution and clustering have the same trend and obey power-law distribution. The higher degree, average path length and average clustering coefficient are, the stronger synthesis operation ability of an entity is.

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References
