Efficient Cyclostationary Based Spectrum Sensing with SLC Diversity over Rayleigh-Shadowing Fading Channels

Ying Zhu\textsuperscript{a,b,*}, Qixun Zhang\textsuperscript{a}, Zhiyong Feng\textsuperscript{a}, Ping Xie\textsuperscript{a}, Ping Zhang\textsuperscript{a}

\textsuperscript{a}Key Laboratory of Universal Wireless Communications, Ministry of Education, Beijing University of Posts and Telecommunications, Beijing 100876, China

\textsuperscript{b}Guilin University of Electronic Technology, Guilin 541004, China

Abstract

Spectrum sensing is a key technical challenge for Cognitive Radio (CR). It is well known that Multi-cycle Cyclostationarity (MC) detector is a powerful method for spectrum sensing. However, conventional MC detector is difficult to implement due to its high computation complexity. This paper pays attention to the fact that the computation complexity can be reduced by simplifying the decision statistic of conventional MC detector. Based on this simplification process, an improved MC detector is proposed. Compared with the conventional one, the proposed detector has low-computational complexity and high-accuracy on sensing performance. Subsequently, Square-law Combining (SLC) diversity reception is introduced to improve the detection capability of proposed detector over composite Rayleigh-shadowing fading channels. The corresponding closed-form average detection probability is derived by using the Moment Generation Function (MGF) approach. Finally, simulation results verify the reliability of our proposed detector, which contributes to a performance gain of approximately 3 dB for 2-branches SLC even if Rayleigh fading and heavy shadowing exist.

Keywords: Spectrum Sensing; Improved MC Detector; Composite Rayleigh-shadowing Fading; SLC; MGF

1 Introduction

With the rapid growth of wireless communications, the available spectrum is becoming overcrowded. To alleviate the spectrum shortage problem, Cognitive Radio (CR) is proposed, which allows the Secondary Users (SUs) to opportunistically access the spectrum of the Primary Users (PUs)
without causing interference [1]. For the purpose of identifying idle spectrum, spectrum sensing is a fundamental task in CR.

The conventional Multi-cycle Cyclostationary (MC) detector is considered as a useful method for spectrum sensing [2]. Although this detector is able to detect weak PU signal due to its robustness to noise uncertainty, it requires extensive computation to provide sufficiently low error probability, causing high complexity [3]. Computational complexity seriously degrades spectrum efficiency of the SU, since all communications should be stopped during detection. Many papers focus on reducing the computation complexity of conventional MC detector. In [4-6], the cooperative and collaborative MC detectors were both introduced to reduce computation complexity. In the proposed cooperative and collaborative scheme, the computation of the decision statistics is done by a group of cooperative SUs in a distributed manner, so that the complexity burden on a single SU is reduced. Nevertheless, a major drawback of these schemes is the need for a very large number of cooperative SUs to make MC detection reliable, which is impractical in CR system sometimes. Therefore, it’s still necessary to improve the conventional MC detector for single SU in local spectrum sensing scheme.

Recently, diversity reception techniques such as Maximum Ratio Combiner (MRC) and Square-law Combining (SLC) are applied to improve the detection reliability in low Signal-to-noise Ratio (SNR) scenario [7][8]. The detection performance of MRC has been analyzed in [7] by using the MGF of fading channel SNR. Study [8] deduces the exact detection performance analysis by utilizing SLC in Rayleigh channels. Since MRC requires complete knowledge of the Channel State Information (CSI), a simpler technique is the SLC, which does not require the CSI.

However, all the above research efforts have focused on Rayleigh fading channel only. In practical implementation, many detectors will be found either in stationary positions or with low mobility in environment with varying degree of vegetation cover [9]. These foliage and temporal stationary behavior introduced another degradation known as shadowing which cannot be mitigated by averaging the received signal strength due to the effect composite Rayleigh-shadowing fading [10]. Hence, if we wish to examine the exact sensing performance of proposed detector, the effect of composite Rayleigh-shadowing fading needs to be investigated. To the best of our knowledge, none of earlier publications has addressed the issue of MC detector over composite Rayleigh-shadowing fading channels.

In summary, the major contributions in this paper include:

1) We propose an improved MC detector, in which the simplified test statistic is presented by a reliable simplification for the test statistic of conventional MC detector. This simplification procedure is able to reduce the computational complexity which caused by computing the test statistic. Simulation results will be shown that it leads to a lower computation complexity while maintaining a sufficient detection sensitivity. From analysis, we find out that the simplified test statistic follows a $\chi^2$ distribution. The closed-form expressions of detection probability and false-alarm probability are then derived.

2) Since complete knowledge of CSI is hard to obtain in practical CR, SLC diversity reception is introduced to improve detection capability and its contribution is demonstrated by comparing it with the case without SLC reception.

3) Based on the SLC diversity detection, the sensing performance of improved MC detector over composite Rayleigh-shadowing fading channels is investigated. The corresponding closed-form average detection probability is derived by using the Moment Generation Function (MGF)
approach. The effect of composite Rayleigh-shadowing fading on sensing performance is demonstrated by simulation results.

The remainder of this paper is organized as follows. In Section II, the improved MC detector is proposed. In Section III, the sensing performance of proposed detector by employing SLC is analysed over composite Rayleigh-shadowing fading channels by using MGF approach. Section IV presents numerical and simulation results and the concluding remarks are given in Section V.

2 Multi-cycle Cyclostationary (MC) Detector

In this section, we start by analyzing the conventional MC detector. Then, we propose an improved MC detector to reduce the computational complexity based on simplifying the test statistic of the conventional MC detector. Subsequently, the closed-form expressions of detection probability and false-alarm probability of proposed detector are derived.

A typical signal detection problem is usually formulated as a binary hypothesis testing problem:

\[
\begin{align*}
H_0 & : r(t) = n(t) \\
H_1 & : r(t) = hs(t) + n(t)
\end{align*}
\] (1)

where \(H_0\) denotes absence of PU, and \(H_1\) denotes the its presence, \(r(t)\) is SU’s received signal, \(h\) is the gain of channel between the PU and the SU, \(n(t)\) is the Additive White Gaussian Noise (AWGN), \(s(t)\) is the PU’s transmitted signal. In spectrum sensing, SU measures the sufficient statistic at first, and then compare it with a threshold which is determined by a desirable false alarm probability to decide between two hypotheses.

2.1 Conventional MC Detector

The sufficient statistic of Maximum Likelihood (ML) detector is given by [11]

\[
Y_{ML} = \int_{t-\frac{T}{2}}^{t+\frac{T}{2}} R_S(u,v) r(u) r^*(v) dv
\] (2)

where \(r(t)\) is received signal, and \(T\) is the observation interval. \(R_S(u,v) = E[s(u)s^*(v)]\) is the autocorrelation function of the transmitted signal \(s(t)\). If the signal is cyclostationary, then the ML detector can be expressed as

\[
Y_{ML} = \sum_{k=1}^{N_\alpha} \int_{-\infty}^{\infty} S_{s}^{\alpha_k}(f)^* S_{r}^{\alpha_k}(f) df, \quad \alpha_k = \frac{k}{T_c}
\] (3)

where \(S_{s}^{\alpha_k}(f)\) and \(S_{r}^{\alpha_k}(f)\) are the Spectral Correlation Functions (SCFs) of \(k\)-th Cycle Frequency (CF) \(\alpha_k = \frac{k}{T_c}\), \(N_\alpha\) is the number of CFs, \(S_{s}^{\alpha_k}(f)\) is the Fourier transform of the cyclic autocorrelation function \(R_{s}^{\alpha_k}(\tau)\), and the time independent function \(R_{s}^{\alpha_k}(\tau)\) is calculated as the Fourier-series coefficient of the periodic autocorrelation function \(R_{s}(t,\tau) = \sum_k R_{s}^{\alpha_k}(\tau)e^{j2\pi\alpha_k t}\). Since the PU’s transmitted signal is unknown, the \(S_{s}^{\alpha_k}(f)\) can be assumed as a rectangular function within \(\Delta f\) bandwidth. Thus,

\[
Y_{MC} = \sum_{k=1}^{N_\alpha} \int_{f-\frac{\Delta f}{2}}^{f+\frac{\Delta f}{2}} S_{r}^{\alpha_k}(v) dv
\] (4)
Consequently, the sum of cyclostationary signal power at all CFs is the sufficient statistic for the optimum detector in ML criterion for cyclostationary signals. This detector is called the MC detector, the total power can be calculated as

$$Y_{MC} = \sum_{k=1}^{N} R_{\alpha_k}^r (\tau = 0)$$  \hspace{1cm} (5)

where $R_{\alpha_k}^r (\tau = 0)$ represents the cyclostationary signal power at the k-th CF.

### 2.2 Improved MC Detector

1) simplified test statistic

Since $Y_{MC} = \text{Re} \left( Y_{MC} \right) + j \text{Im} \left( Y_{MC} \right)$ is a complex random variable, the test statistic of conventional MC detector can be given by [6]

$$T_{MC} = |Y_{MC}|^2 = \sum_{k=1}^{N} |R_{\alpha_k}^r (\tau = 0)|^2 + \sum_{k=1}^{N} \sum_{n=1,n \neq k}^{N} R_{\alpha_k}^r (\tau = 0) R_{\alpha_n}^s (\tau = 0)$$  \hspace{1cm} (6)

For reducing computational complexity, we will make a simplification to $T_{MC}$. Since the second term of $T_{MC}$ brings a large amount of computation, here it is omitted. The first term of $T_{MC}$ is used as the test statistic of the proposed MC detector. Thus, the simplified test statistic of proposed detector can be defined as

$$T_{sim} = \sum_{k=1}^{N} |R_{\alpha_k}^r (\tau = 0)|^2$$  \hspace{1cm} (7)

2) computational complexity analysis

In order to investigate the computational complexity of $T_{MC}$ and $T_{sim}$, we make analysis as follows:

- For computing $T_{MC}$: in (6), we need to do complex multiplication for $N^2_{\alpha}$ times and complex addition for $(N^2_{\alpha} - 1)$ times to compute $T_{MC}$, therefore, the total computation of $T_{MC}$ is $2N^2_{\alpha} - 1$, let $\Theta(N^2_{\alpha})$ denote the computational complexity of conventional detector.

- For computing $T_{sim}$: in (7), we need to do complex multiplication for $N_{\alpha}$ times and complex addition for $(N_{\alpha} - 1)$ times to compute $T_{sim}$, thus, the total computation of $T_{sim}$ is $2N_{\alpha} - 1$, denote $\Theta(N_{\alpha})$ as the computational complexity of proposed detector.

Since $N_{\alpha} \gg 1$, it can be clearly seen that the computational complexity is greatly reduced by the simplification of test statistic, which means the proposed MC detector is of lower computational complexity than conventional one.

Next, the sensing performance of proposed detector will be analyzed. By simulation results in Section IV, we will verify that there is no appreciable difference in detection performance between the proposed detector and conventional detector.

3) detection probability and false-alarm probability of proposed detector over AWGN channel

Since $T_{sim}$ is the test statistic of proposed detector, the structure of detector can be defined as

$$T_{sim} = \sum_{k=1}^{N} |R_{\alpha_k}^r (\tau = 0)|^2 \begin{cases} H_1 > \lambda \\ < H_0 \end{cases}$$  \hspace{1cm} (8)
where \( \lambda \) is detection threshold.

Consequently, the false-alarm and detection probability of proposed detector can be represented as

\[
P_f = \int_{\lambda}^{+\infty} P(T_{sim} \mid H_0) dT_{sim}
\]

\[
P_d = \int_{\lambda}^{+\infty} P(T_{sim} \mid H_1) dT_{sim}
\]

where \( \lambda \) is detection threshold. In order to obtained the conditional probability density functions (pdfs) \( P(T_{sim} \mid H_0) \) and \( P(T_{sim} \mid H_1) \), we should focus on the \( R_{\alpha k}(\tau = 0) \) in (7) first. Denote

\[
Y_1 = R_{\alpha k}(\tau = 0)
\]

The cyclic autocorrelation function of \( Y_1 \) can be shown by

\[
R_{\alpha k}^{\gamma}(\tau) = \lim_{T \to \infty} \int_{-T/2}^{T/2} r(t + \tau) r^*(t - \tau) e^{-j2\pi\alpha_k t} dt
\]

From (11) and (12), the discrete-time counterpart of \( Y_1 \) is given by

\[
Y_1 = \frac{1}{N_S} \sum_{w=1}^{N_S} |r[w]|^2 e^{-j2\pi\alpha_k w}
\]

Hence, the \( H_0 \) and \( H_1 \) according to (1) can be represented as

\[
\begin{align*}
H_0 : Y_1 &= \frac{1}{N_S} \sum_{w=1}^{N_S} |n[w]|^2 e^{-j2\pi\alpha_k w} \\
H_1 : Y_1 &= \frac{1}{N_S} \sum_{w=1}^{N_S} |s[w] + n[w]|^2 e^{-j2\pi\alpha_k w}
\end{align*}
\]

where \( n[w] = \text{Re} (n[w]) + j\text{Im} (n[w]) \) is a complex an Additive White Gaussian Noise (AWGN) sample with zero mean and variance \( \sigma_0^2 \). \( s[w] \) is a PU signal sample. The proposed MC detector can be viewed as the measure of signal power for a single CF \( \alpha_k \), then the structure of detector can be defined as

\[
T_1 = |Y_1|^2 = |R_{\alpha k}^{\gamma}(\tau = 0)|^2 > \frac{H_1}{H_0} \lambda
\]

Therefore, for the case of detection \( \alpha_k \), the detection and false-alarm probability can be represented as

\[
P_{f,\alpha_k} = \int_{\lambda}^{\infty} P(T_1 \mid H_0) dT_1
\]

\[
P_{d,\alpha_k} = \int_{\lambda}^{\infty} P(T_1 \mid H_1) dT_1
\]

The Probability Density Functions (PDFs) \( P(T_1 \mid H_0) \) and \( P(T_1 \mid H_1) \) can be derived from \( P(Y_1 \mid H_0) \) and \( P(Y_1 \mid H_1) \). Base on the central limit theorem for large \( N_S, N_\alpha >> 1 \), \( Y_1 \) is approximately Gaussian. Since \( E \{|n[w]|^2\} = \sigma_0^2 \) and \( \text{var} \{|n[w]|^2\} = \text{var} \{\text{Re}^2 (n[w]) + \text{Im}^2 (n[w])\} = \sigma_0^4 \), the mean and variance of \( Y_1 \) under \( H_0 \) are

\[
E \{Y_1 \mid H_0\} = \frac{\sigma_0^2}{N_S} \sum_{w=1}^{N_S} e^{-j2\pi\alpha_k w} = 0
\]
\[
\text{var} \{Y_1 | H_0\} = \frac{1}{N_S} \sum_{w=1}^{N_S} |e^{-j2\pi\alpha_kw}|^2 \text{var} \{|n[w]|^2\} = \frac{\sigma_d^4}{N_S} \tag{19}
\]

Since \(\{Y_1 | H_0\} = \text{Re}(Y_1 | H_0) + j\text{Im}(Y_1 | H_0)\) is a complex Gaussian random variable, then \((T_1 | H_0) = \{Y_1 | H_0\}^2 = [\text{Re}(Y_1 | H_0)]^2 + [\text{Im}(Y_1 | H_0)]^2\) follows a central \(\chi^2\) distribution with two degrees of freedom, which pdf can be expressed as

\[
P(T_1 | H_0) = \frac{1}{2\sigma_1^2} e^{-\frac{T_1}{2\sigma_1^2}}, \quad \sigma_1^2 = \frac{\sigma_0^4}{N_S}
\tag{20}
\]

Thus, the false alarm probability for detection \(\alpha_k\) is

\[
P_{f,\alpha_k} = e^{-\frac{\lambda}{2\sigma_1^2}} \tag{21}
\]

Similarly, since \(E\{|s[w] + n[w]|^2\} = |s[w]|^2 + \sigma_0^2\), the mean of \((Y_1 | H_1)\) is

\[
E \{Y_1 | H_1\} = \frac{1}{N_S} \sum_{w=1}^{N_S} (|s[w]|^2 + \sigma_0^2)e^{-j2\pi\alpha_kw} = \frac{1}{N_S} \sum_{w=1}^{N_S} |s[w]|^2 e^{-j2\pi\alpha_kw} = P_{\alpha_k} \tag{22}
\]

where the complex-valued \(P_{\alpha_k}\) is the signal power at \(\alpha_k\). Since noise samples are statistically independent, the variance of \((Y_1 | H_1)\) is

\[
\text{var} \{Y_1 | H_1\} = \frac{1}{N_S} \sum_{w=1}^{N_S} |e^{-j2\pi\alpha_kw}|^2 \text{var} \{|s[w]|^2 + |n[w]|^2\} = \frac{2\sigma_0^2 P}{N_S} + \frac{\sigma_0^4}{N_S} \tag{23}
\]

where \(P = \frac{1}{N_S} \sum_{w=1}^{N_S} |s[w]|^2\). Since the \((T_1 | H_1) = \{Y_1 | H_1\}^2 = [\text{Re}(Y_1 | H_1)]^2 + [\text{Im}(Y_1 | H_1)]^2\) is non-central \(\chi^2\) distribution with two degrees of freedom, its pdf is given by

\[
P(T_1 | H_1) = \frac{1}{2\sigma_2^2} e^{-\frac{T_1+\alpha_1}{2\sigma_2^2}} I_0 \left(\frac{\sqrt{T_1}u_1}{\sigma_2}\right) \tag{24}
\]

where \(u_1 = \sqrt{\{E[\text{Re}(Y_1 | H_1)]\}^2 + \{E[\text{Im}(Y_1 | H_1)]\}^2} = |P_{\alpha_k}| \) and \(\sigma_2^2 = 2\sigma_0^2 P/N_S + \sigma_0^4/N_S\). Then, the corresponding detection probability can be given by

\[
P_{d,\alpha_k} = Q_1 \left(\frac{u_1}{\sigma_2}, \sqrt{\frac{\lambda}{\sigma_2}}\right) \tag{25}
\]

where \(Q_1(\cdot, \cdot)\) is the generalized Marcum-Q function.

The derivation above shows that the detection and false-alarm probability of proposed detector when it measures a single CF \(\alpha_k\). However, the test statistic \(T_{\text{sim}} = \sum_{k=1}^{N_\alpha} |R_{r,k}^{\alpha_k}(\tau = 0)|^2 = \sum_{k=1}^{N_\alpha} T_1\), which means there are \(N_\alpha\) different CFs should be measured by proposed detector. Since \(T_1\) follows a central \(\chi^2\) distribution with two degrees of freedom under \(H_0\) and a non-central \(\chi^2\) distribution with two degrees of freedom under \(H_1\), it can be easily concluded that \(T_{\text{sim}}\) follows a central \(\chi^2\) distribution with \(2N_\alpha\) degrees of freedom under \(H_0\) and a non-central \(\chi^2\) distribution with \(2N_\alpha\) degrees of freedom under \(H_1\). Therefore, the conditional pdfs of \(T_{\text{sim}}\) can be represented as

\[
P(T_{\text{sim}} | H_0) = \frac{1}{2\sigma_1^2 N_\alpha^2 \Gamma(N_\alpha)} e^{-\frac{T_{\text{sim}}}{2\sigma_1^2}} T_{\text{sim}}^{N_\alpha-1}, \quad \sigma_1^2 = \frac{\sigma_0^4}{N_S} \tag{26}
\]
\[ P(T_{\text{sim}} | H_1) = \frac{1}{2\sigma_2^2} e^{-\frac{T_{\text{sim}} + u_2}{2\sigma_2^2}} (\frac{T_{\text{sim}}}{u_2})^{\frac{N_a - 1}{2}} I_{N_a - 1}(\frac{\sqrt{T_{\text{sim}} u_2}}{\sigma_2^2}), \quad \sigma_2^2 = \frac{2\sigma_0^2 P_{\text{n}} + \sigma_0^2}{N_a} \] (27)

where \( u_2 = \sum_{k=1}^{N_a} u_1 = \sum_{k=1}^{N_a} |P_{\alpha_k}| \), \( I_v(\cdot) \) is the th-order modified Bessel function of the first kind and \( \Gamma(\cdot) \) is the gamma function. The corresponding false alarm and detection probabilities of proposed detector are

\[
P_f = \int_{\lambda}^{\infty} P(T_{\text{sim}} | H_0) dT_{\text{sim}} = \frac{\Gamma \left( N_a, \frac{\lambda}{2\sigma_0^2} \right)}{\Gamma(N_a)} \] (28)

\[
P_d = \int_{\lambda}^{\infty} P(T_{\text{sim}} | H_1) dT_{\text{sim}} = Q_{N_a} \left( \frac{\sqrt{u_2}}{\sigma_2}, \sqrt{\lambda} \right) \] (29)

Since the proposed detector is in AWGN channel (i.e. \( h_i = 1 \)), \( u_2 = \sum_{k=1}^{N_a} |P_{\alpha_k}| \) is signal power and \( \sigma_0^2 \) is noise power, let \( \gamma \) denote SNR, then \( \gamma = \frac{u_2}{\sigma_0^2} \). Thus, (29) can be rewritten as

\[
P_d = Q_{N_a} \left( \frac{\sigma_0 \sqrt{\gamma}}{\sigma_2}, \frac{\sqrt{\lambda}}{\sigma_2} \right) \] (30)

\section{Sensing Performance with SLC Diversity Reception over Composite Rayleigh-Shadowing Fading Channels}

In this section, SLC diversity reception is utilized to improve detection reliability of the proposed detector. Then, the sensing performance over composite Rayleigh-shadowing fading channels is investigated.

\subsection{SLC Diversity Reception}

In SLC diversity reception, the outputs of the square-law devices (square-and-integrate operation per branch) are added to yield a new test statistic \( T_{\text{sim}} = \sum_{i=1}^{L} T_{\text{sim},i} \), where \( T_{\text{sim},i} \) denotes the test statistic from the \( i \)-th square-law device and \( L \) is the number of diversity branches.

Under \( H_1 \), \( T_{\text{sim}} \) has a non-central \( \chi^2 \) distribution with \( LN_a \) degrees of freedom, and non-centrality parameter of \( \gamma_i = \sum_{i=1}^{L} \gamma_i \). Thus, the detection probability can be obtained by

\[
P_{d,\text{SLC}} = Q_{LN_a} \left( \frac{\sigma_0 \sqrt{\gamma_i}}{\sigma_2}, \frac{\sqrt{\lambda}}{\sigma_2} \right) \] (31)

\subsection{Average Detection Probability with Composite Rayleigh-Shadowing Fading}

The generalized Marcum-Q function can be written as a circular contour integral within the contour radius \( r \in [0, 1) \). Therefore, expression (31) can be rewritten as

\[
P_{d,\text{SLC}} = \frac{e^{-\frac{\lambda}{2\sigma_2^2}}}{2\pi j} \int_{\Delta} e^{\frac{\sigma_0^2}{2} (\frac{1}{2} - 1) \gamma_i + \frac{\lambda}{2\sigma_2^2} z} z^{LN_a} (1 - z)^{-1} dz \] (32)
where $\Delta$ is a circular contour of radius $r \in [0, 1]$.

The Moment Generating Function (MGF) of average received SNR $\overline{\gamma}_t$ is $M_{\overline{\gamma}_t}(s) = E(e^{-s\overline{\gamma}_t})$, where $E(\cdot)$ means expectation. Thus, the average detection probability, $P_{d,SLC}$, is given by

$$P_{d,SLC} = \frac{e^{-\frac{\lambda}{2\pi^2}}} {2\pi j} \oint_{\Delta} f(z)dz$$

(33)

where $f(z) = M_{\overline{\gamma}_t} \left( \frac{\sigma_0^2}{2\sigma_2^2} (1 - \frac{1}{z}) \right) e^{\frac{x^2}{2\sigma_2^2}} \frac{1}{z^{\epsilon_n}(1-z)}$.

1) Over Rayleigh fading channels

The MGF of Rayleigh fading combined SLC is $M_{\overline{\gamma}_t,SLC}^\text{Ray}(s) = (1 + \overline{\gamma}_t s)^{-\lambda}$. After substitute this MGF in (30), the average detection probability, $P_{d,SLC}$, under Rayleigh fading can be written in the form of expression (33) with $f(z) = \frac{e^{\frac{x^2}{2\sigma_2^2}}}{(1+\mu)^{f}(z^{\epsilon_n}(1-z))^{2\beta}}$, where $\mu = \sigma_0^2 \overline{\gamma}_t / 2\sigma_2^2$ and $\beta = L(N_\alpha - 1)$.

In radius $r \in [0, 1]$, there are $\beta$ poles at the origin $z = 0$ and $L$ poles at $z = \mu/(1+\mu)$. By applying the residue theorem to (33), detection probability under Rayleigh fading is obtained as

$$\frac{P_{d,SLC}^\text{Ray}}{P_{d,SLC}} = \begin{cases} 
\frac{e^{-\frac{\lambda}{2\pi^2}} \left[ \text{Res} \left( f; 0, \beta \right) + \text{Res} \left( f; \frac{\mu}{1+\mu}, L \right) \right]} {\text{Res} \left( f; \frac{\mu}{1+\mu}, L \right)} & N_\alpha > 1 \\
\frac{e^{-\frac{\lambda}{2\pi^2}} \text{Res} \left( f; \frac{\mu}{1+\mu}, L \right)} {\text{Res} \left( f; \frac{\mu}{1+\mu}, L \right)} & N_\alpha = 1 
\end{cases}$$

(34)

where $\text{Res} \left( f; 0, \beta \right) = \frac{D^{\beta-1} \left( \frac{x^{\epsilon_n}}{(1+\mu)^{\beta}} \right)}{(1+\mu)^{\beta}(\beta-1)!} \bigg|_{z=0}$ and $\text{Res} \left( f; \frac{\mu}{1+\mu}, L \right) = \frac{D^{L-1} \left( \frac{x^{\epsilon_n}}{(1+\mu)^{L-1}} \right)}{(1+\mu)^{L}(L-1)!} \bigg|_{z=\frac{\mu}{1+\mu}}$.

$D^n (f(z))$ denotes the $n$th derivative of $f(z)$ with respect to $z$.

2) Over composite Rayleigh-shadowing fading channels

Since shadowing process is typically modeled as a lognormal distribution, the composite Rayleigh-shadowing channels model follows a gamma-lognormal distribution as [9]

$$f_{\text{Com}}(x) = \sum_{i=1}^{N} \phi_i e^{-\epsilon_i x}, \quad x \geq 0, \phi_i \geq 0, \epsilon_i \geq 0$$

(35)

where $\phi_i = \rho_i e^{-(\sqrt{2}\delta\eta_i+\psi)}/(\sqrt{\pi} \sum_{i=1}^{N} \rho_i), \epsilon_i = e^{-(\sqrt{2}\delta\eta_i+\psi)}, N$ is the number of terms in the mixture, $\eta_i$ and $\rho_i$ are abscissas and weight factors for the Gaussian-Laguerre integration, $\psi$ and $\delta$ are the mean and the standard deviation of the lognormal distribution, respectively. The MGF of composite Rayleigh-shadowing combined SLC is given as $M_{\overline{\gamma}_t,SLC}^\text{Com}(s) = \sum_{i=1}^{N} \left( \frac{\phi_i}{\epsilon_i + \psi} \right)^L$. Thus, the average detection probability with composite Rayleigh-shadowing fading can be evaluated in closed-form as

$$\frac{P_{d,SLC}^\text{Com}}{P_{d,SLC}} = \begin{cases} 
\frac{e^{-\frac{\lambda}{2\pi^2}} \sum_{i=1}^{N} \left( \frac{\phi_i}{\epsilon_i} \right)^L} {\sum_{i=1}^{N} \left( \frac{\phi_i}{\epsilon_i} \right)^L} \times \left[ \text{Res} \left( f; 0, \beta \right) + \text{Res} \left( f; \frac{\mu}{1+\mu}, L \right) \right] & N_\alpha > 1 \\
e^{-\frac{\lambda}{2\pi^2}} \sum_{i=1}^{N} \left( \frac{\phi_i}{\epsilon_i} \right)^L \text{Res} \left( f; \frac{\mu}{1+\mu}, L \right) & N_\alpha = 1 
\end{cases}$$

(36)
where \( f_i(z) = \frac{2^{z-1}}{(1+\mu_i)^{z}} \left( z - \frac{\mu_i}{1+\mu_i} \right)^{z-1}(1-z) \), \( \mu_i = \sigma_i^2 / 2 \sigma_i^2 \epsilon_i \). Following a similar procedure as in Rayleigh fading case, the residues \( \text{Res} (f_i; 0, \beta) \) and \( \text{Res} \left( f_i; \frac{\mu_i}{1+\mu_i}, L \right) \) can be obtained, we omit the expressions here for brevity.

4 Numerical Results

In this section, we provide simulation results to demonstrate the sensing performance of the proposed detector over composite Rayleigh-shadowing fading channels. We first verify the reliability of our proposed detector. Subsequently, the sensing performance of proposed detector over composite Rayleigh-shadowing fading channels is investigated. The shadowing effects considered here were introduced by Loo in [10] and are (i) light shadowing \((\psi = 0.115 \text{ and } \delta = 0.115)\), which corresponds to sparse tree cover and (ii) heavy shadowing \((\psi = 3.914 \text{ and } \delta = 0.806)\), which corresponds to dense tree cover.

Fig. 1 shows the Receiver Operating Characteristic (ROC) curves (plots of detection probability \( P_d \) versus false alarm probability \( P_f \)) of proposed MC detector and conventional MC detector over AWGN channel for different SNRs. It can be observed that there is no appreciable difference on sensing performance between the proposed and conventional detector, due to the simplification to the test statistic of conventional detector. Although this simplification slightly degrades sensing accuracy, it still maintains a satisfactory sensing capability. In all, Fig. 1 and the analysis of computational complexity (in Section II.B.2) demonstrated that the proposed MC detector can reduce the computational complexity while still maintaining sufficient detection sensitivity.

Fig. 1: ROC curves for the proposed and conventional MC detector with different SNRs (\( \gamma = 5, 10, 15 \) dB)

Fig. 2 depicts the ROC curves of proposed detector in composite Rayleigh-shadowing fading environment without SLC diversity reception. We take \( N = 10 \) in (35), which makes the Mean Square Error (MSE) between the exact gamma-lognormal channel model and the approximated mixture gamma channel model in (32) less than \( 10^{-4} \). The numerical results match well with their theoretical analysis, confirming the accuracy of the analysis. It can be seen that the performance
of proposed detector degrades with increase in shadowing, and improves at higher SNR. Further, for Rayleigh and composite Rayleigh-light shadowing environments, there is a slight difference in detector’s performance. However, there is a significant performance degradation of the proposed detector due to the heavy shadowing effect in lower average SNR (e.g. 5 dB). Next, we consider the effect of SLC reception in alleviating the composite Rayleigh-shadowing.

![ROC curves of proposed detector over Rayleigh and composite Rayleigh-shadowing fading channels](image1)

**Fig. 2:** ROC curves of proposed detector over Rayleigh and composite Rayleigh-shadowing fading channels

Fig. 3 demonstrated that SLC diversity reception improves the detection performance, even with the composite Rayleigh-shadowing fading. For example, in Rayleigh-heavy shadowing fading environment with the low average SNR (e.g. 5 dB), we find that the detection probability for a 4-branches SLC case ($L=4$) is almost six times than that for a 1-branch non-SLC case ($L=1$). Furthermore, for a 2-branches SLC case ($L=2$), there is approximately 3 dB performance gain compared with non-SLC case. Therefore, the SLC mitigates the impact of fading-shadowing and introduces a significant improvement in the detection probability.

![$P_d$ Vs. average SNR with SLC over Rayleigh and composite Rayleigh-shadowing fading channels](image2)

**Fig. 3:** $P_d$ Vs. average SNR with SLC over Rayleigh and composite Rayleigh-shadowing fading channels ($P_f=0.01$, $L=1, 2, 4$)
5 Conclusion

In this paper, we first propose an improved MC detector. Compared with the conventional MC detector, the proposed method enables reducing the computational complexity while maintaining sufficient detection sensitivity. The corresponding closed-form expressions of the average detection probability and false-alarm probability of proposed detection are derived. By employing SLC diversity, the detection performance of proposed detector with composite Rayleigh-shadowing fading is investigated. Simulation results demonstrated that the proposed detector is efficient and accurate, furthermore, the proposed detector based SLC spectrum sensing technique results in a significant detector performance gain over composite Rayleigh-shadowing fading channels in low SNR regime.

References